

## Unit [ 4 ] : Geometry And Measurement

### Lesson [ 1 ] : Geometric Concepts – The Relations Between Angles

#### 1 The line segment

It is a set of points consisting of two distinct points and all points between them when we join them by a ruler.

#### 2 The straight line

If we extend the line segment in both directions infinitely , we will get a straight line.

#### 3 The ray

It is a line segment extended from only one of its terminals without limit.

#### Remarks

- Each of line segment , straight line and ray is an infinite set of points.
- $\overline{AB} \subset \overrightarrow{AB}$  ,  $\overrightarrow{AB} \subset \overleftrightarrow{AB}$       i.e.  $\overline{AB} \subset \overrightarrow{AB} \subset \overleftrightarrow{AB}$

#### 4 The plane

A plane is a flat and unbounded surface , and it is extended without limit in all directions

#### 5 The angle

It is the union of two rays with the same starting point , and this point is called the vertex of the angle , and the two rays are called the two sides of the angle.

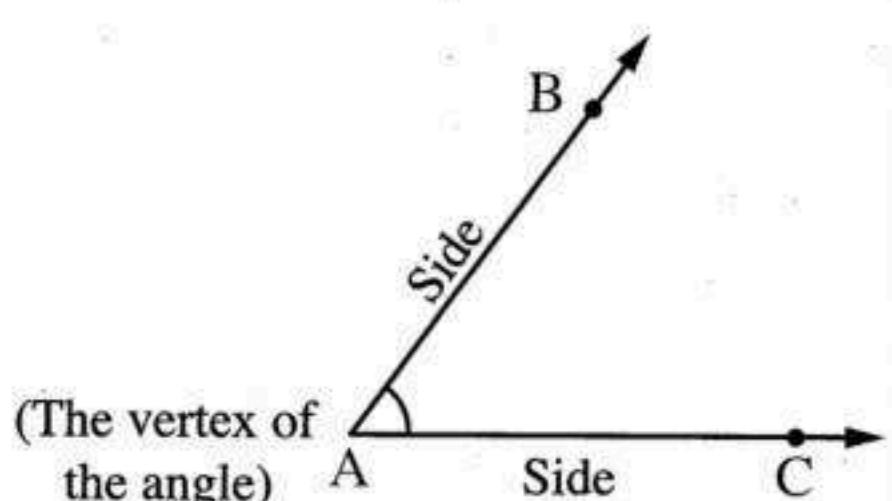
For example :

In the opposite figure :

$\overrightarrow{AB}$  and  $\overrightarrow{AC}$  are two rays having the same starting point A , then:

$$\overrightarrow{AB} \cup \overrightarrow{AC} = \text{the angle } CAB$$

\* A is the vertex of the angle CAB



#### Measurement of the angle

- A degree is divided into parts smaller than it , and they are minute (') and second (") where the degree equals 60 minutes and the minute equals 60 second and we can change the units of measuring angle by using the calculator.

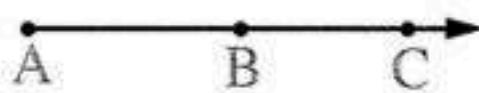
For example :

$$23\frac{1}{2}^\circ = 23^\circ 30' , 65\frac{1}{4}^\circ = 65^\circ 15' , 81\frac{1}{8}^\circ = 81^\circ 7' 30''$$

## The types of angles

The angles are classified according to their measures as follows :

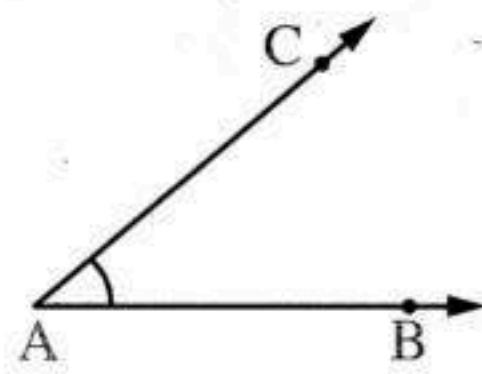
### 1 Zero angle



Its measure =  $0^\circ$

Its sides are coincident.

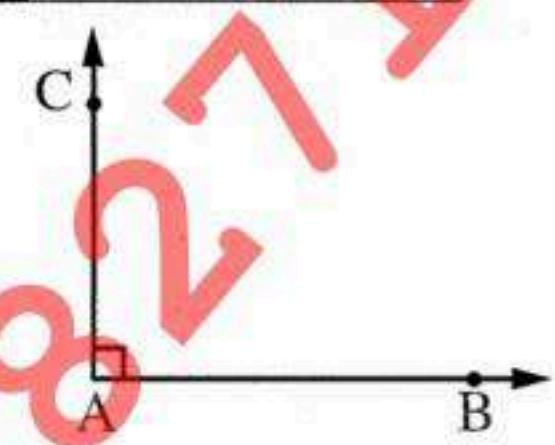
### 2 Acute angle



Its measure is more than  $0^\circ$  and less than  $90^\circ$

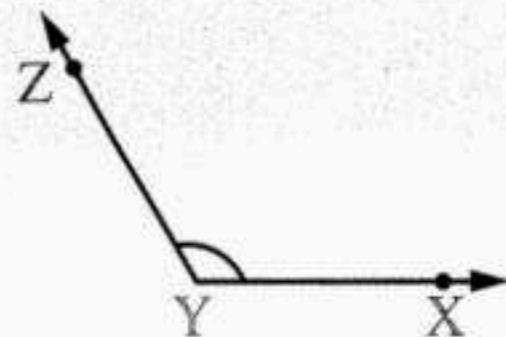
i.e.  $0^\circ < \text{measure of acute angle} < 90^\circ$

### 3 Right angle



Its measure =  $90^\circ$

### 4 Obtuse angle

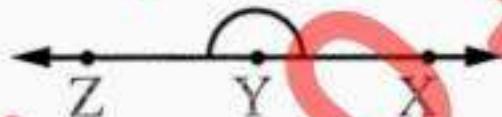


Its measure is more than  $90^\circ$  and less than  $180^\circ$

i.e.

$90^\circ < \text{measure of obtuse angle} < 180^\circ$

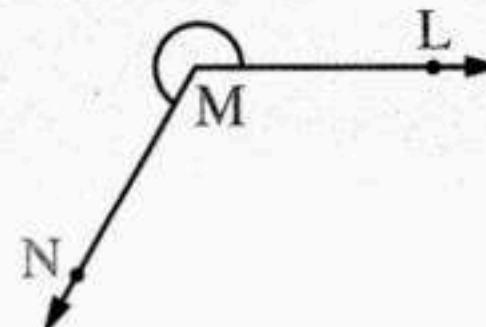
### 5 Straight angle



Its measure =  $180^\circ$

Its sides are forming one straight line.

### 6 Reflex angle



Its measure is more than  $180^\circ$  and less than  $360^\circ$

i.e.

$180^\circ < \text{measure of reflex angle} < 360^\circ$

### Remark : -

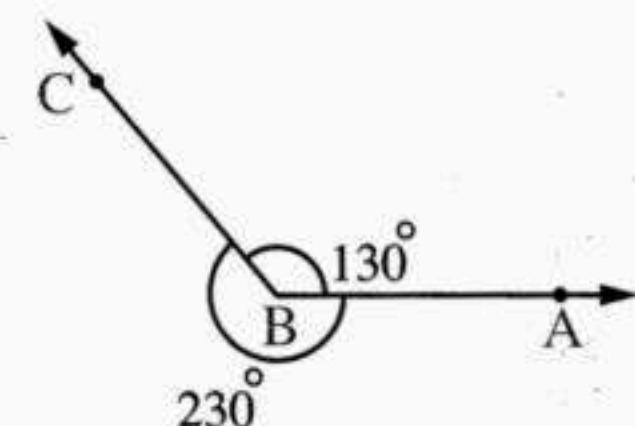
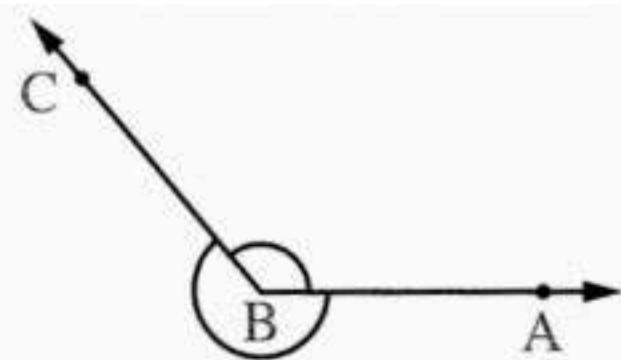
- In the opposite figure :

$$m(\angle ABC) + m(\text{reflex } \angle ABC) = 360^\circ$$

For example :

If  $m(\angle ABC) = 130^\circ$

$$\begin{aligned} \text{, then } m(\text{reflex } \angle ABC) \\ = 360^\circ - 130^\circ = 230^\circ \end{aligned}$$



## Some relations between the angles

### Adjacent angles

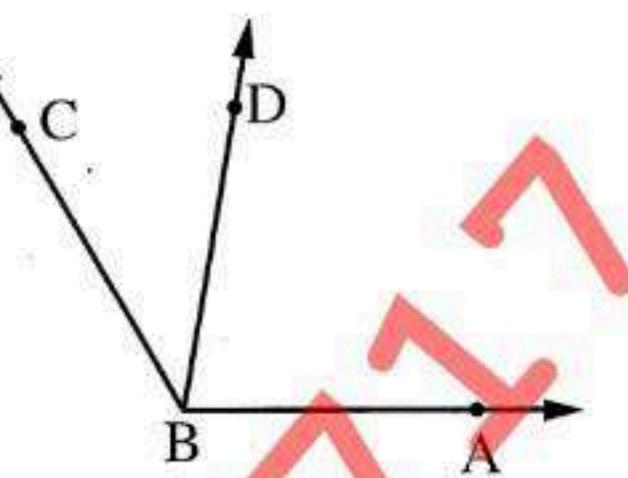
Two angles are said to be adjacent if they have a common vertex and a common side and the other two sides are on opposite sides of this common side.

**For example :**

• In the opposite figure :

∠ ABD and ∠ DBC are two adjacent angles , for :

- They have a common vertex B and a common side  $\overrightarrow{BD}$
- The two other sides  $\overrightarrow{BA}$  and  $\overrightarrow{BC}$  are on two opposite sides of  $\overrightarrow{BD}$



### Complementary angles

Two angles are said to be complementary if the sum of their measures is  $90^\circ$

**For example :**

The two angles whose measures are  $55^\circ$  and  $35^\circ$  are called two complementary angles because  $55^\circ + 35^\circ = 90^\circ$

### Remarks

- 1 The two complementary angles are either acute angles or one of them is zero angle and the other is a right angle.
- 2 The complements of the same angle (or the equal angles in measure) are equal in measure.  
i.e. If ∠ A complements ∠ B , ∠ C complements ∠ B , then  $m(\angle A) = m(\angle C)$

### Supplementary angles

Two angles are said to be supplementary if the sum of their measures is  $180^\circ$

**For example :**

The two angles whose measures are  $143^\circ$  and  $37^\circ$  are called two supplementary angles because  $143^\circ + 37^\circ = 180^\circ$

### Remarks

- 1 The two supplementary angles are either one of them is obtuse and the other is acute or each of them is a right angle or one of them is zero angle and the other is a straight angle.
- 2 The supplements of the same angle (or the equal angles in measure) are equal in measure.  
i.e. If ∠ A supplements ∠ B and ∠ C supplements ∠ B , then  $m(\angle A) = m(\angle C)$

### The two adjacent supplementary angles

Two adjacent angles formed by a straight line and a ray with a starting point on this straight line , are supplementary.

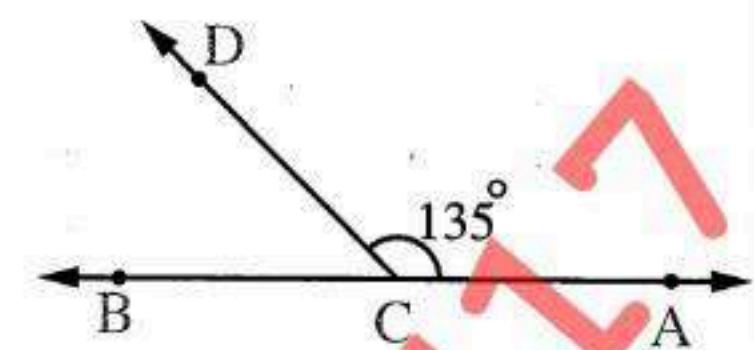
i.e. In the opposite figure :

If  $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{C\}$

Therefore ,  $m(\angle ACD) + m(\angle DCB) = 180^\circ$  "Straight angle"

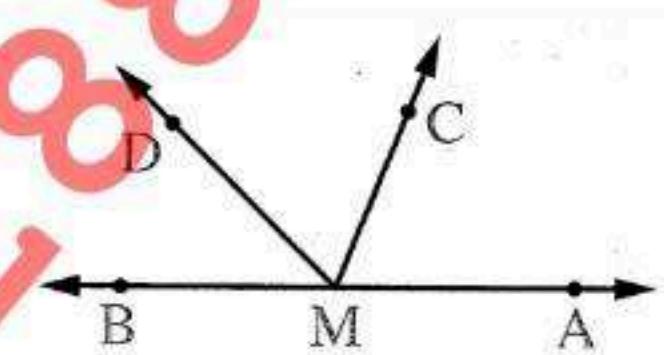
And if  $m(\angle ACD) = 135^\circ$

Then  $m(\angle DCB) = 180^\circ - 135^\circ = 45^\circ$



**Remark :**

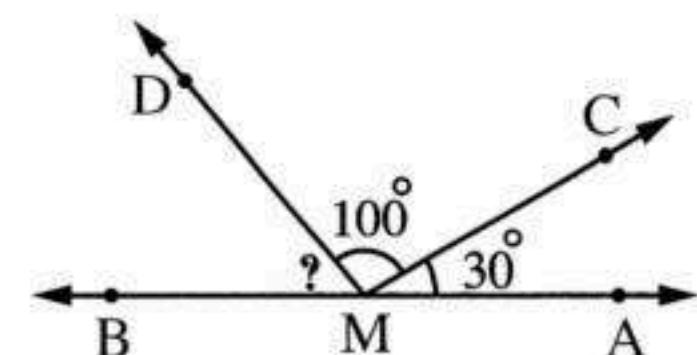
If  $M \in \overleftrightarrow{AB}$  , and  $\overrightarrow{MC}$  and  $\overrightarrow{MD}$  are drawn on one side of  $\overleftrightarrow{AB}$  , then  $m(\angle AMC) + m(\angle CMD) + m(\angle DMB) = 180^\circ$



**For example :**

• In the opposite figure :

If  $M \in \overleftrightarrow{AB}$  ,  $m(\angle AMC) = 30^\circ$   
 $, m(\angle CMD) = 100^\circ$   
 $, \text{then} : m(\angle DMB) = 180^\circ - (30^\circ + 100^\circ) = 180^\circ - 130^\circ = 50^\circ$



### Vertically opposite angles (V.O.A.)

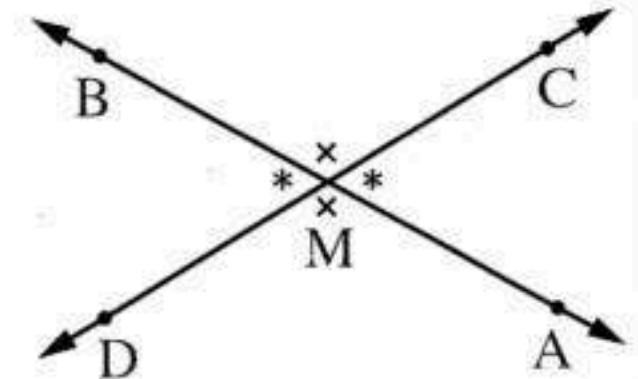
If two straight lines intersect , then the measures of each two vertically opposite angles are equal.

• In the opposite figure :

If  $\overleftrightarrow{AB}$  and  $\overleftrightarrow{CD}$  intersect at M

Then :

- $\angle AMC$  and  $\angle BMD$  are vertically opposite angles ,  
 $, \text{then } m(\angle AMC) = m(\angle BMD)$
- Also ,  $\angle CMB$  and  $\angle AMD$  are vertically opposite angles ,  
 $, \text{then } m(\angle CMB) = m(\angle AMD)$



### Accumulative angles at a point

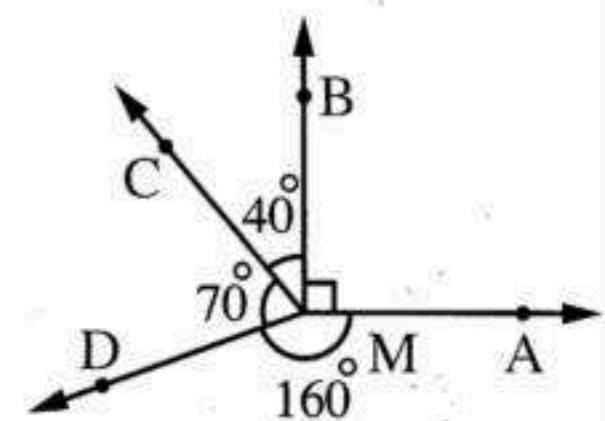
The sum of the measures of the accumulative angles at a point is  $360^\circ$

• In the opposite figure :

$\overrightarrow{MA}$  ,  $\overrightarrow{MB}$  ,  $\overrightarrow{MC}$  and  $\overrightarrow{MD}$  are rays having the same starting point M

The angles  $\angle AMB$  ,  $\angle BMC$  ,  $\angle CMD$  and  $\angle DMA$  are called accumulative angles at the point M , hence we get :

$$m(\angle AMB) + m(\angle BMC) + m(\angle CMD) + m(\angle DMA) = 90^\circ + 40^\circ + 70^\circ + 160^\circ = 360^\circ$$



## The angle bisector

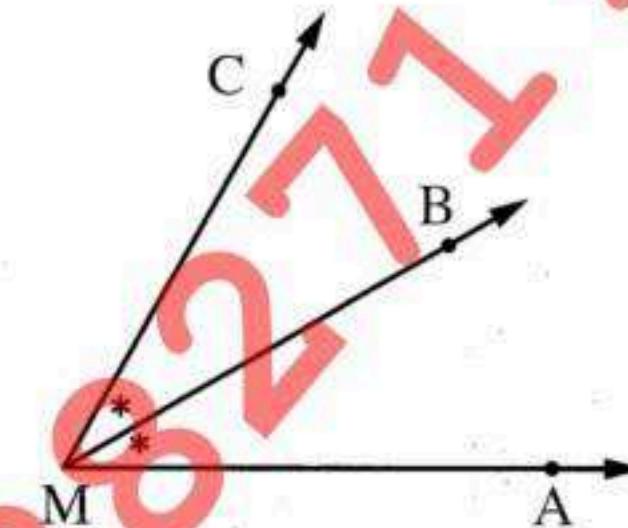
It is the ray that divides the angle into two halves (two equal angles in measure).

- In the opposite figure :

$\overrightarrow{MB}$  bisects  $\angle AMC$

i.e.  $m(\angle AMB) = m(\angle BMC) = \frac{1}{2} m(\angle AMC)$

or  $m(\angle AMC) = 2 m(\angle AMB) = 2 m(\angle BMC)$



## Lesson [ 3 ] : Congruence - Congruent Triangles

### First Congruence of two line segment

Generally

Two line segments are congruent if they are equal in length.

If the length of  $\overline{XY} =$  the length of  $\overline{ZL}$ , then  $\overline{XY} \equiv \overline{ZL}$

### Second Congruence of two angles

Generally

Two angles are congruent if they are equal in measure.

If  $m(\angle C) = m(\angle D)$ , then  $\angle C \equiv \angle D$

### Third Congruence of two polygons

Remark

If the two polygons are congruent , then each side and each angle in one of them is congruent to its corresponding element in the other polygon.

For example :

If ABC and XYZ are two triangles in which :

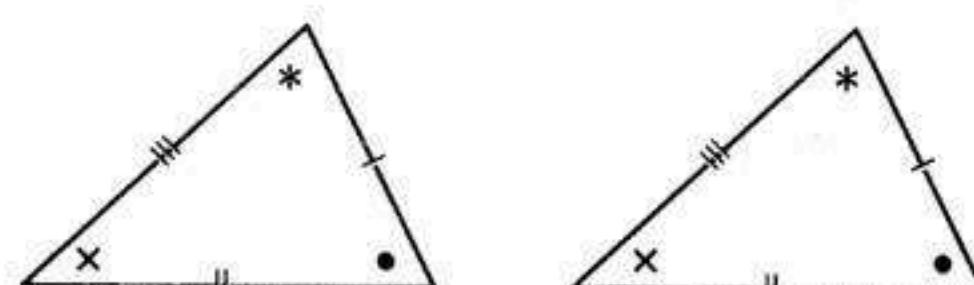
1  $AB = XY$  ,  $AC = XZ$

and  $BC = YZ$

2  $m(\angle A) = m(\angle X)$  ,  $m(\angle B) = m(\angle Y)$

and  $m(\angle C) = m(\angle Z)$

, then :  $\Delta ABC \equiv \Delta XYZ$



### Cases of congruence of two triangles

Two sides and the included angle

Two angles and one side

Three sides

Hypotenuse and one side in the right-angled triangle

#### The first case (Two sides and the included angle S.A.S.)

Two triangles are congruent if two sides and the included angle of one triangle are congruent to the corresponding parts of the other triangle.

##### Remark

In the case of congruence of two triangles by two sides and the included angle , the included angle should be between the two sides.

#### The second case (Two angles and one side A.S.A.)

Two triangles are congruent if two angles and the side drawn between their vertices of one triangle are congruent to the corresponding parts of the other triangle.

#### The third case (Three sides S.S.S.)

Two triangles are congruent if each side of one triangle is congruent to the corresponding side of the other triangle.

##### Remark

If each angle of one triangle is congruent to the corresponding angle of the other triangle , it is not necessary for the two triangles to be congruent.

#### The fourth case (Hypotenuse and one side in the right-angled triangle R.H.S.)

Two right-angled triangles are congruent if the hypotenuse and a side of one triangle are congruent to the corresponding parts of the other triangle.

##### Remark

The two right-angled triangles are congruent if the two sides of the right angle in one of them are congruent to the corresponding elements in the other triangle. (This case is an application of the first case of congruence of two triangles).

## Lesson [ 4 ] : Parallelism

If a straight line intersects two parallel straight lines , then each two alternate angles are equal in measure.

If a straight line intersects two parallel straight lines , then each two corresponding angles are equal in measure.

If a straight line intersects two parallel straight lines , then each two interior angles in the same side of the transversal are supplementary.

## How to prove that two straight lines are parallel ?

The two straight lines are parallel if a third straight line intersects them (as a transversal) and one of the following cases is satisfied :

- 1** Two alternate angles have the same measure.
- 2** Two corresponding angles have the same measure.
- 3** Two interior angles in the same side of the transversal are supplementary.

- 3** Using the geometric instruments or computer , draw the straight lines  $L_1$  ,  $L_2$  ,  $L_3$  and  $L_4$  , then draw the transversal  $M_1$  to cut them at A , B , C and D respectively

Where :  $AB = BC = CD$

Then draw another transversal  $M_2$  to cut them at E , F , G and H respectively.

, then find by measuring the lengths of  $\overline{EF}$  ,  $\overline{FG}$  and  $\overline{GH}$

We find that :  $EF = FG = GH$

**Generally :** If :  $L_1 // L_2 // L_3 // L_4$  ,  $M_1$  and  $M_2$  are two transversal in which  $AB = BC = CD$  , then  $EF = FG = GH$

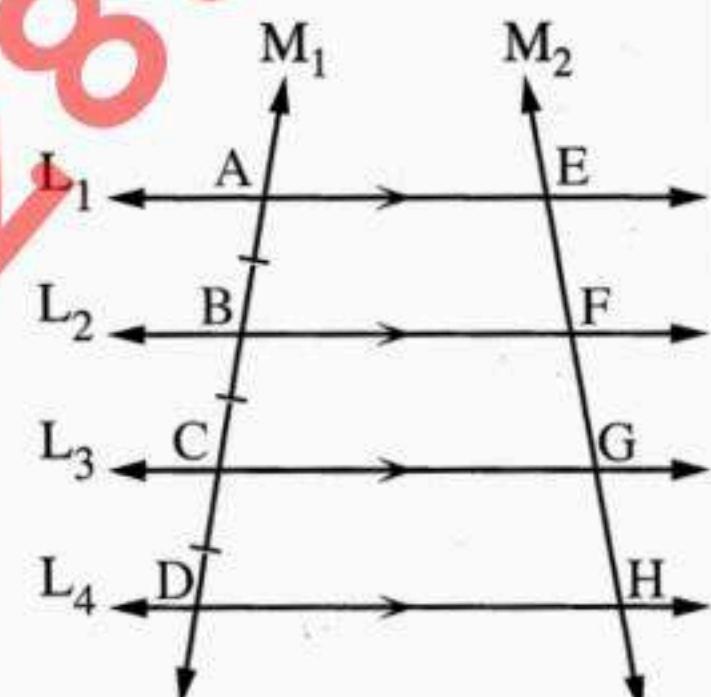
### Generally

If parallel straight lines divide a straight line into segments of equal lengths , then they divide any other straight line into segments of equal lengths.

**Generally :** The perpendicular to one of two coplaner parallel straight lines is perpendicular to the other

*And vice versa* , if two coplaner straight lines are perpendicular to a third one , then the two straight lines are parallel.

**Generally :** If two straight lines are parallel to a third straight line , then these two straight lines are parallel.



## Lesson [ 5 ] : Geometric Constructions

### First Constructing a perpendicular from a point outside a straight line :

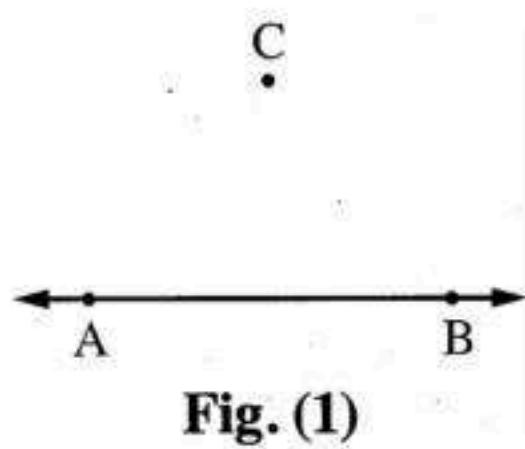


Fig. (1)

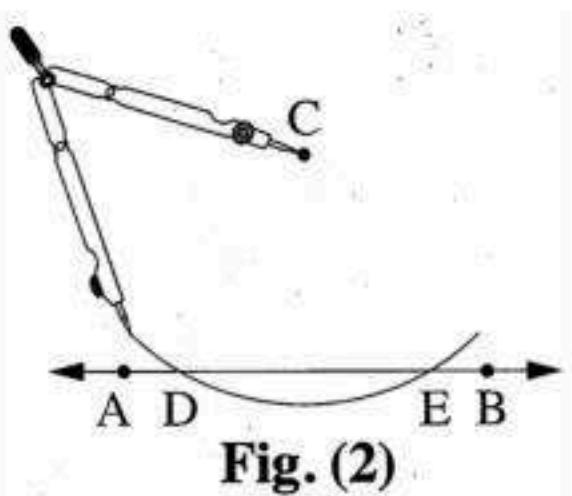


Fig. (2)

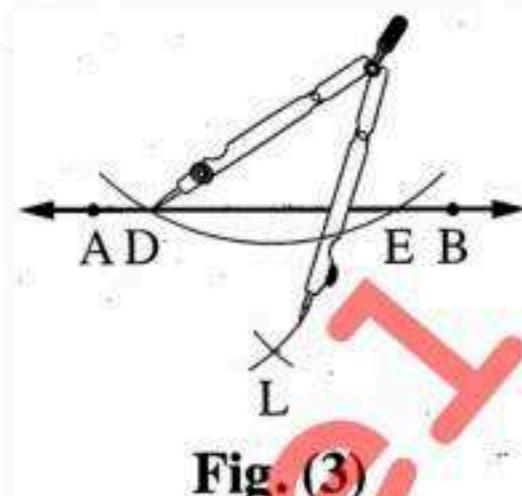


Fig. (3)

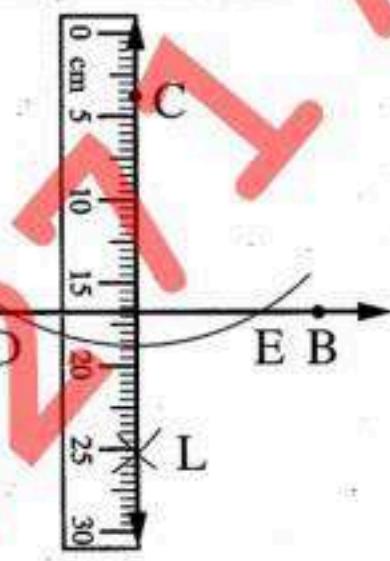


Fig. (4)

### Second Drawing a perpendicular to a straight line that passes through a point which belongs to that straight line.

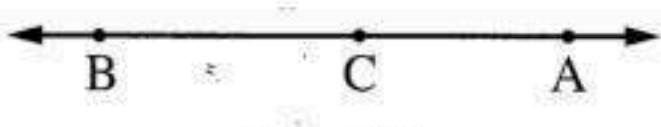


Fig. (1)

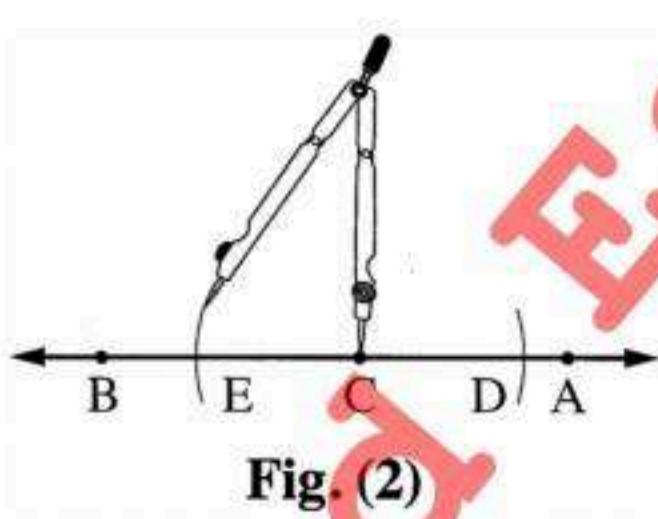


Fig. (2)

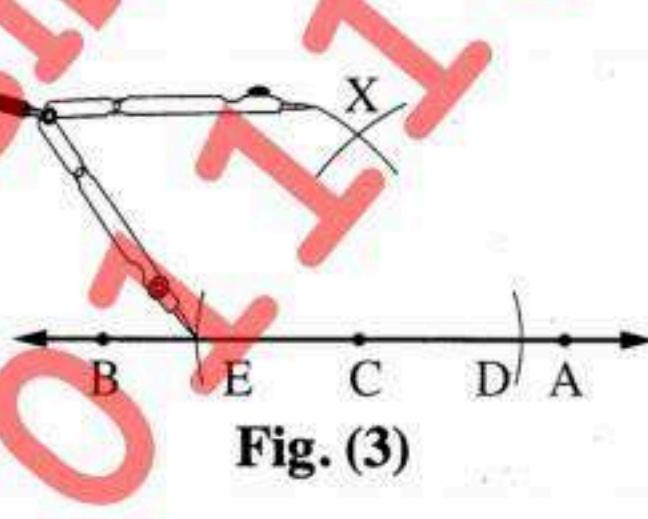


Fig. (3)

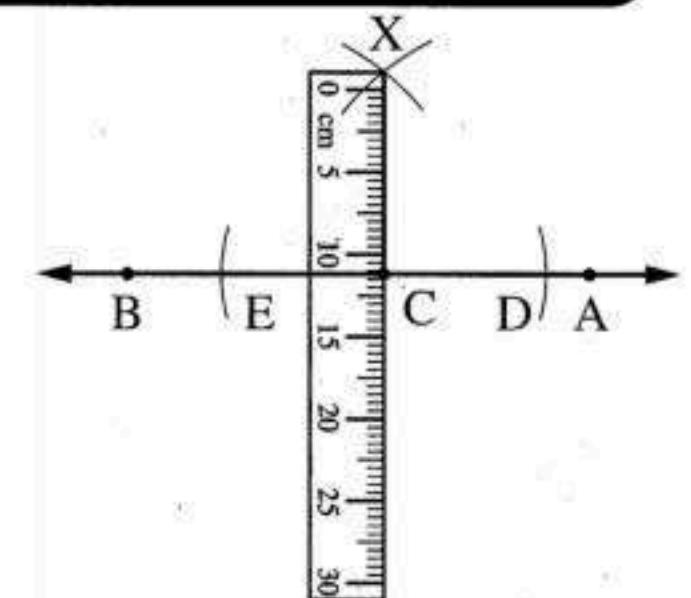


Fig. (4)

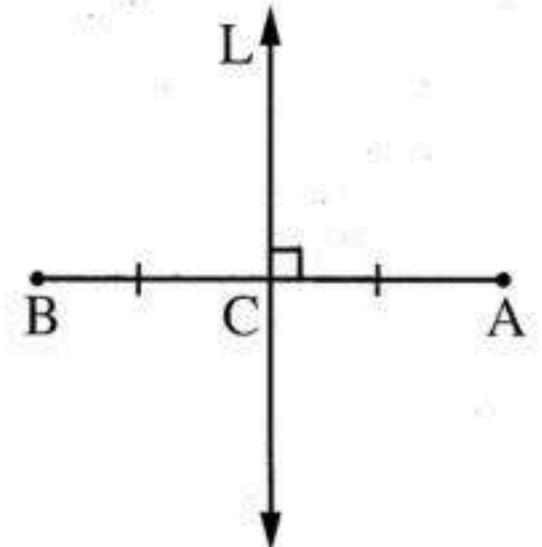
### The axis of symmetry of a line segment :

It is the straight line perpendicular to it from its midpoint.

- In the opposite figure :

If C is the midpoint of  $\overline{AB}$  and the straight line  $L \perp \overline{AB}$  from the point C

Then the straight line L is the axis of symmetry of the line segment  $\overline{AB}$



### Third Bisecting a given line segment “Constructing the symmetry axis of a given line segment”



Fig. (1)

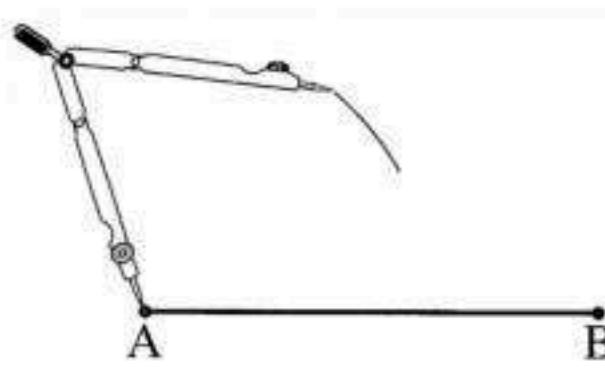


Fig. (2)

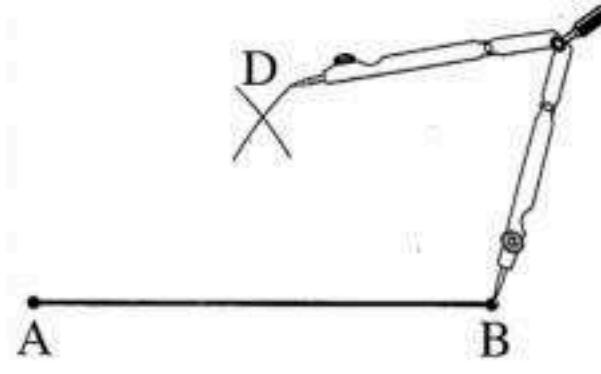


Fig. (3)

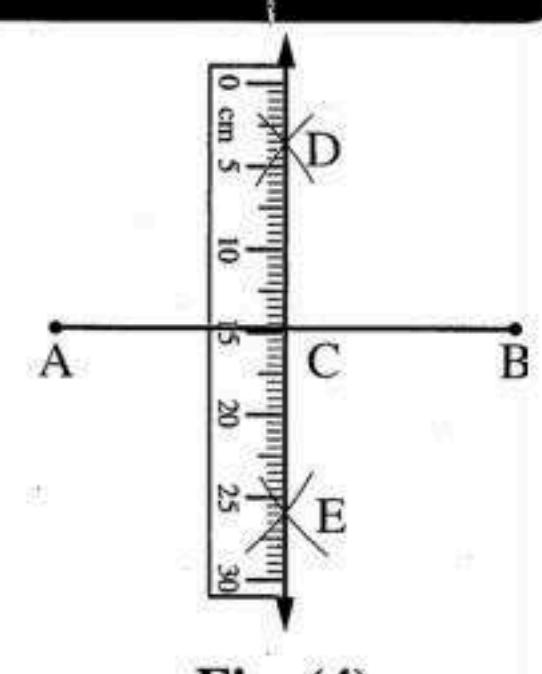
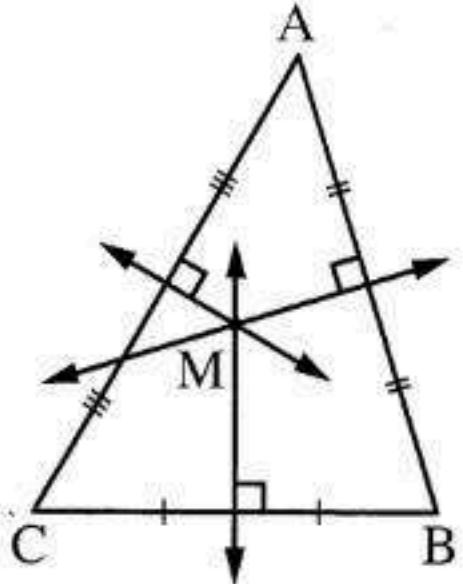
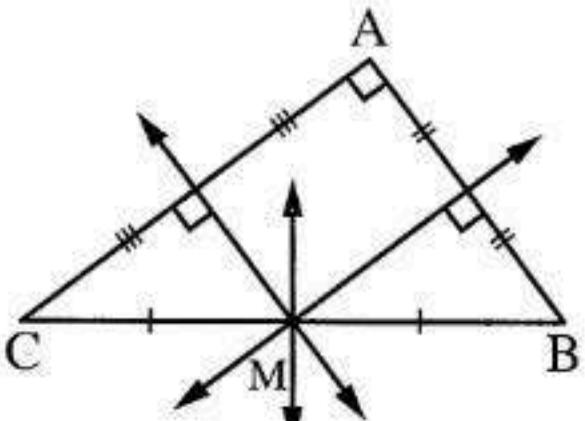
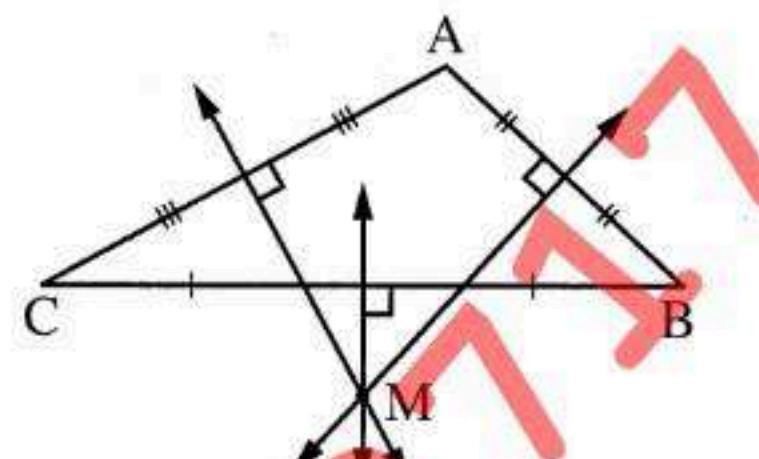


Fig. (4)

### Remarks

The axes of symmetry of the sides of any triangle are intersecting at one point (say M).

The position of M differs according to the type of the triangle as follows :

Acute-angled triangle	Right-angled triangle	Obtuse-angled triangle
		
M is inside the triangle.	M is the midpoint of the hypotenuse.	M is outside the triangle.

• The lengths of the line segments joining the point of intersection of the axes of symmetry and the vertices of the triangle are equal in all previous cases.  
i.e.  $AM = BM = CM$

#### Fourth Constructing the bisector of a given angle :

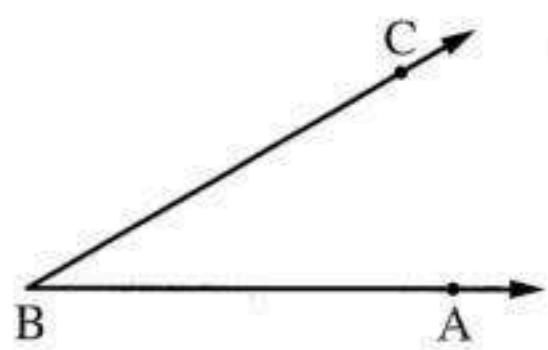


Fig. (1)

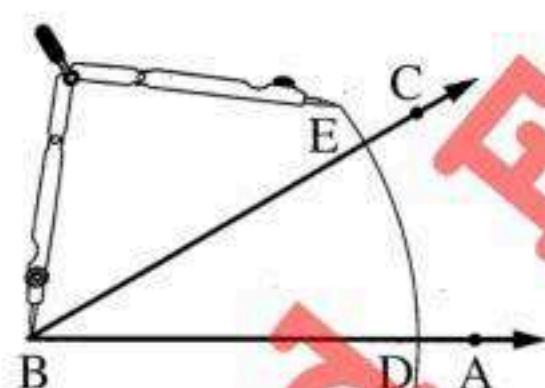


Fig. (2)

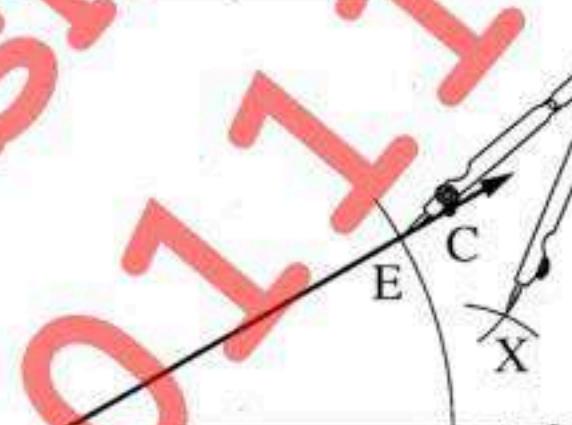


Fig. (3)

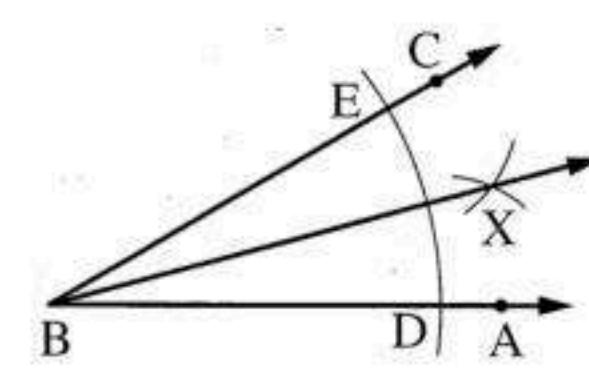


Fig. (4)

#### Fifth Constructing an angle to be congruent to a given angle (without using protractor) :

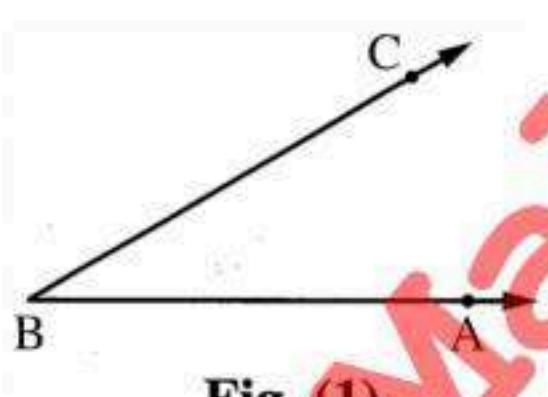


Fig. (1)

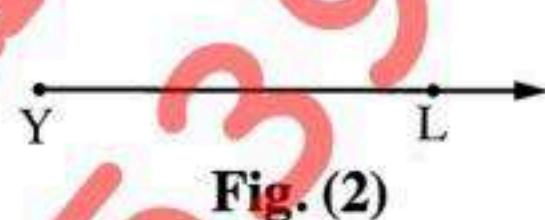


Fig. (2)

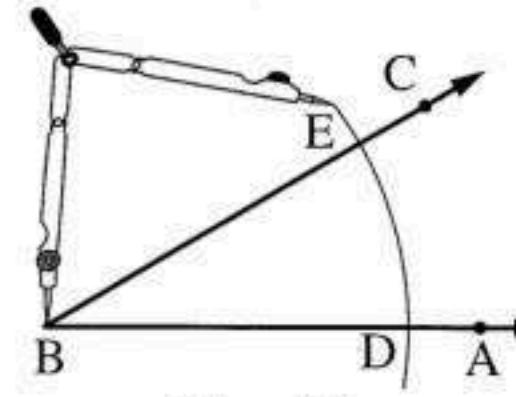


Fig. (3)

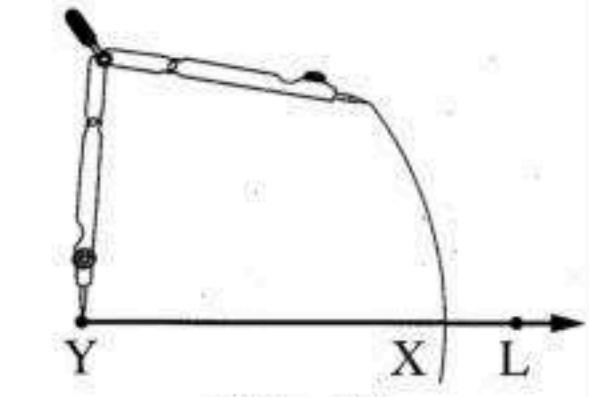


Fig. (4)

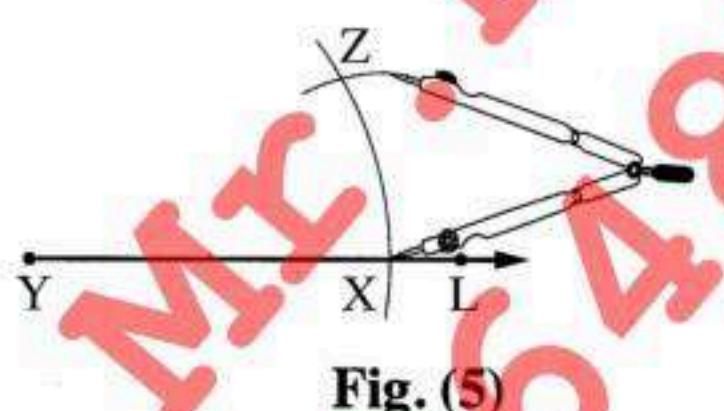


Fig. (5)

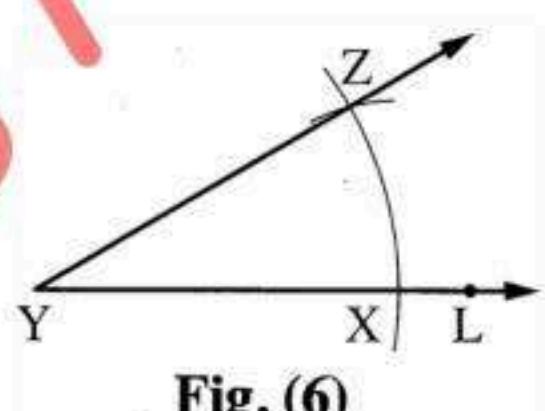


Fig. (6)

#### Sixth Drawing a straight line from a given point parallel to given straight line.

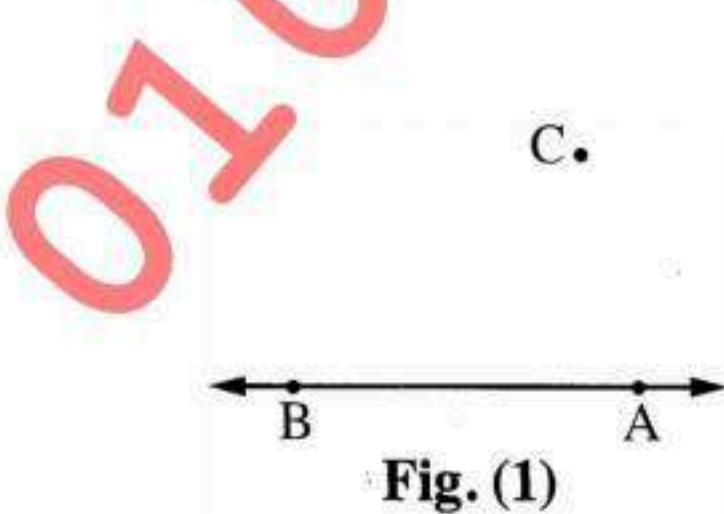


Fig. (1)

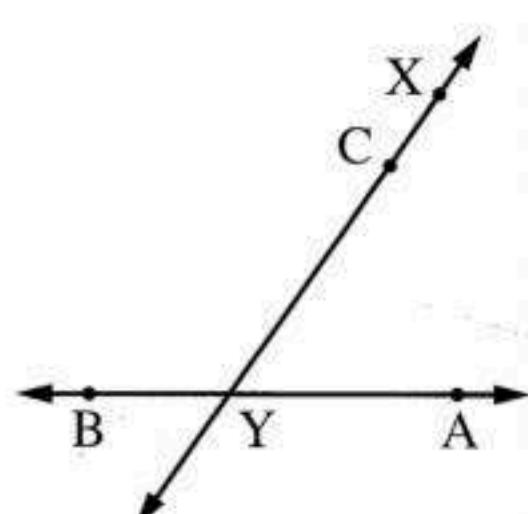


Fig. (2)

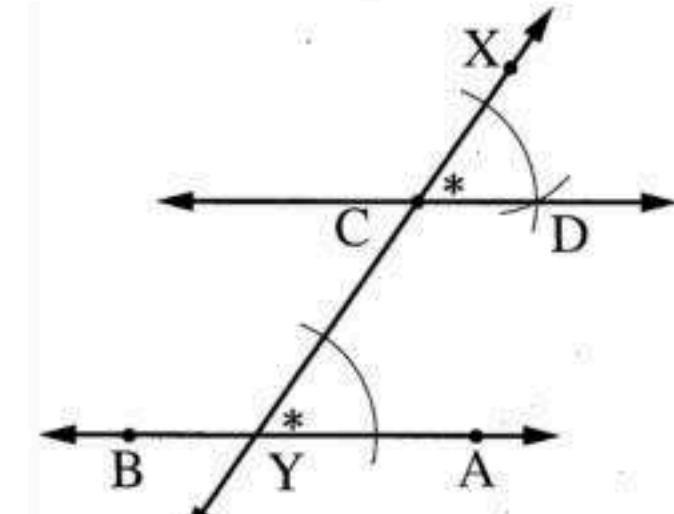


Fig. (3)

**[ A ] : Choose The Correct Answer : -**

1	The measure of the right angle = ..... °				A
1	(a) 90	(b) 180	(c) 270	(d) 360	
2	The measure of the straight angle = ..... °				B
2	(a) 90	(b) 180	(c) 360	(d) zero	
3	The type of the angle of measure $179^\circ 60'$ is ..... angle.				C
3	(a) acute	(b) obtuse	(c) straight	(d) right	
4	The angle whose measure is $108^\circ$ is ..... angle.				C
4	(a) an acute	(b) a right	(c) an obtuse	(d) a straight	
5	The angle whose measure is $210^\circ$ is ..... angle.				D
5	(a) an acute	(b) a right	(c) an obtuse	(d) a reflex	
6	If $m(\angle B) = 120^\circ$ , then $m(\text{reflex } \angle B) =$ .....				C
6	(a) 60	(b) 120	(c) 240	(d) 180	
7	$\overline{AB} \dots \overrightarrow{AB}$				C
7	(a) $\in$	(b) $\notin$	(c) $\subset$	(d) $\not\subset$	
8	If $m(\angle A) + m(\angle B) = 90^\circ$ , then $\angle A$ , $\angle B$ are ..... angles.				A
8	(a) complementary	(b) supplementary	(c) equal	(d) adjacent	
9	The angle of measure $70^\circ$ complements an angle of measure ..... °				B
9	(a) 90	(b) 20	(c) 180	(d) 110	
10	If $\angle A$ complements $\angle B$ , $m(\angle A) = m(\angle B)$ , then $m(\angle A) =$ ..... °				C
10	(a) 90	(b) 180	(c) 45	(d) 60	
11	The acute angle complements ..... angle.				A
11	(a) an acute	(b) an obtuse	(c) a right	(d) a reflex	
12	If the two adjacent angles are complementary , then their outer sides are .....				A
12	(a) perpendicular.	(b) coincident.	(c) on the same straight line.	(d) skew.	

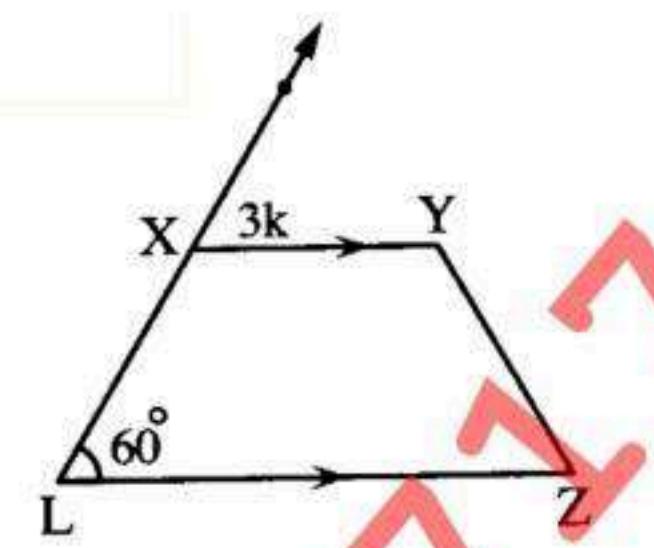
13	The two angles $35^\circ$ , $55^\circ$ are ..... (a) complementary. (b) supplementary. (c) adjacent. (d) reflex.				A
14	If $m(\angle X) = 2 m(\angle Y)$ , $\angle X$ and $\angle Y$ are two complementary angles , then $m(\angle Y) =$ ..... (a) $90^\circ$ (b) $45^\circ$ (c) $30^\circ$ (d) $15^\circ$				C
15	The supplementary angle of the angle of measure $70^\circ$ is ..... (a) $30^\circ$ (b) $110^\circ$ (c) $20^\circ$ (d) $290^\circ$				B
16	The acute angle supplements ..... angle. (a) an acute (b) an obtuse (c) a right (d) a reflex				B
17	If one of two supplementary angles is right. then the other is ..... angle. (a) an acute (b) a right (c) an obtuse (d) a straight				B
18	The obtuse angle supplements ..... angle. (a) an acute (b) an obtuse (c) a right (d) a reflex				A
19	If $\angle A$ supplements $\angle B$ and $\angle A \equiv \angle B$ , then $m(\angle A) =$ ..... <sup>°</sup> (a) 180 (b) 90 (c) 360 (d) 45				B
20	The sum of the measures of two adjacent angles formed by a straight line and a ray with a starting point on this straight line is ..... (a) $90^\circ$ (b) $180^\circ$ (c) $270^\circ$ (d) $360^\circ$				B
21	If $\angle A$ and $\angle B$ are supplementary angles and $m(\angle A) = 2 m(\angle B)$ , then $m(\angle A) =$ ..... <sup>°</sup> (a) 90 (b) 60 (c) 180 (d) 120				D
22	If the ratio between two adjacent supplementary angles is $2 : 3$ , then the measure of the smallest angle is ..... <sup>°</sup> (a) 108 (b) 36 (c) 72 (d) 125				C
23	If $\angle A \equiv \angle B$ , $\angle A$ and $\angle B$ are two supplementary angles , then $\frac{1}{3} m(\angle A) =$ ..... (a) $15^\circ$ (b) $30^\circ$ (c) $40^\circ$ (d) $60^\circ$				B

24	The sum of measures of the accumulative angles at a point equals ..... (a) $90^\circ$ (b) $180^\circ$ (c) $630^\circ$ (d) $360^\circ$				D
25	If $AB = XY$ , then $\overline{AB} \dots \overline{XY}$ (a) $>$ (b) $\equiv$ (c) $<$ (d) $\neq$				B
26	In $\triangle ABC$ , if $m(\angle A) = 30^\circ$ , $m(\angle B) = 90^\circ$ , then $m(\angle C) = \dots$ (a) $60^\circ$ (b) $30^\circ$ (c) $45^\circ$ (d) $90^\circ$				A
27	If $\triangle XYZ \cong \triangle LMN$ , then $m(\angle Y) = m(\angle \dots)$ (a) L      (b) M      (c) N      (d) X				B
28	If $\triangle ABC \cong \triangle XYZ$ and $m(\angle C) = 50^\circ$ , then $m(\angle \dots) = 50^\circ$ (a) X      (b) Y      (c) Z      (d) M				C
29	If $\overline{AB} \equiv \overline{XY}$ , then $AB - XY = \dots$ (a) AB      (b) XY      (c) 1      (d) zero				D
30	If $\triangle ABC \cong \triangle XYZ$ , then $BC = \dots$ (a) YZ      (b) XZ      (c) XY      (d) AC				A
31	If $\triangle ABC \cong \triangle MNO$ , $m(\angle M) = 40^\circ$ and $m(\angle C) = 60^\circ$ , then $m(\angle B) = \dots^\circ$ (a) 40      (b) 80      (c) 60      (d) 100				B
32	If $\triangle ABC \cong \triangle XYZ$ and $m(\angle A) + m(\angle X) = 100^\circ$ , then $m(\angle A) = \dots^\circ$ (a) 100      (b) 80      (c) 40      (d) 50				D
33	If $\triangle ABC \cong \triangle XYZ$ , $m(\angle A) + m(\angle C) = 110^\circ$ , then $m(\angle Y) = \dots^\circ$ (a) 50      (b) 70      (c) 80      (d) 100				B
34	If two straight lines are parallel to a third straight line , then they are ..... (a) perpendicular.    (b) intersecting.    (c) parallel.    (d) congruent.				C
35	If parallel straight lines divide a straight line into segments of equal lengths , then they divide any other straight line into segments of ..... lengths. (a) parallel      (b) not equal      (c) equal      (d) perpendicular				C

36	The straight line that is perpendicular to one of two parallel lines is ..... to the other.	C
(a) parallel      (b) congruent      (c) perpendicular      (d) equal		
37	If $\overleftrightarrow{AB} \parallel \overleftrightarrow{XY}$ , then $\overleftrightarrow{AB} \cap \overleftrightarrow{XY} = \dots$	C
(a) $\{B\}$ (b) $\overline{AX}$ (c) $\emptyset$ (d) $\{Y\}$		
38	If $L_1 \parallel L_2$ and $L_1 \perp L_3$ , then .....	D
(a) $L_1 \perp L_2$ (b) $L_1 \parallel L_3$ (c) $L_2 \parallel L_3$ (d) $L_2 \perp L_3$		
39	In the opposite figure : $m(\angle CMB) = \dots^\circ$	C
(a) 230      (b) 100      (c) 130      (d) 30		
40	In the opposite figure : The necessary condition to make $\Delta ABC \cong \Delta DEF$ is .....	C
(a) $AB = DE$ (b) $AC = DF$ (c) $BC = EF$ (d) $m(\angle A) = m(\angle D)$		
41	In the opposite figure : $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{C\}$ , then $x = \dots^\circ$	C
(a) 180      (b) 30      (c) 20      (d) 120		
42	In the opposite figure : If $AB = DE$ , $BC = EC$ , $\overline{AC} \perp \overline{BD}$ , then $m(\angle A) = \dots$	B
(a) $m(\angle B)$ (b) $m(\angle D)$ (c) $m(\angle DEC)$ (d) $m(\angle ACD)$		

- 43

**In the opposite figure : If  $\overline{XY} \parallel \overline{LZ}$  , then  $k = \dots$**



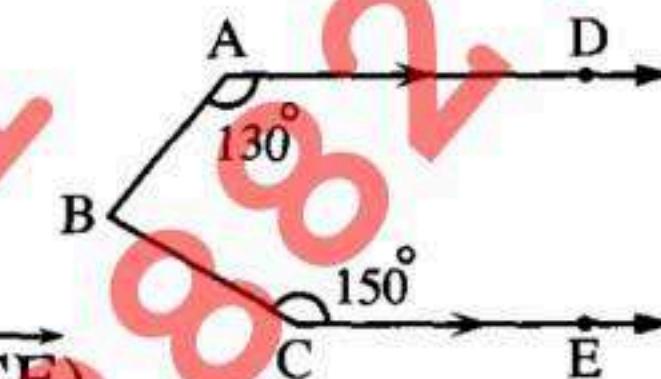
- 44

**In the opposite figure :**

$$\overrightarrow{AD} \parallel \overrightarrow{CE}$$

$$, m(\angle B) = \dots \text{ } ^\circ$$

(Hint : Draw a line passing through B and parallel to  $\overrightarrow{AD}$  and  $\overrightarrow{CE}$ )

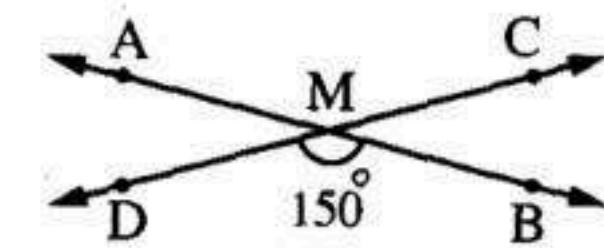


- 45

**In the opposite figure :**

$$\overleftrightarrow{AB} \cap \overleftrightarrow{DC} = \{M\}, m(\angle AMC) = \dots$$

- (a) 30                          (b) 210                          (c) 150

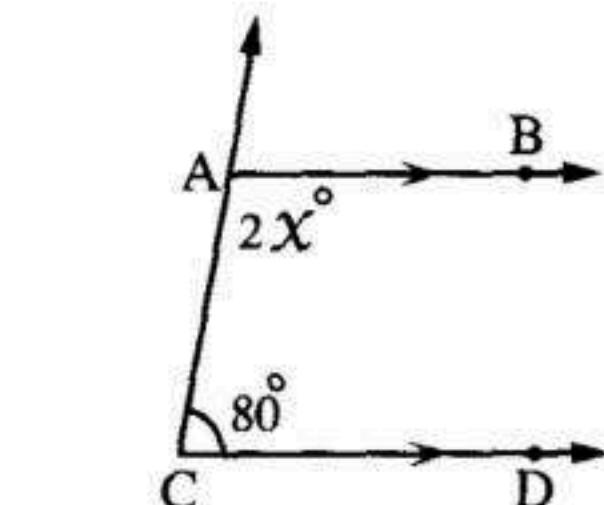


- 46

### **In the opposite figure :**

$$\chi = \dots \text{.}$$

- (a) 40 (b) 80 (c) 50

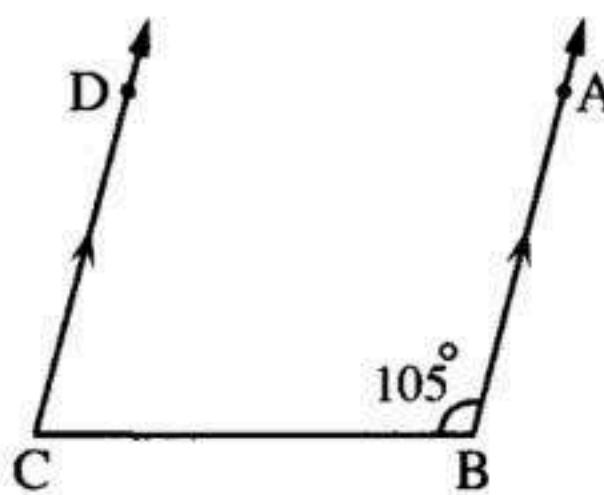


- 47

**In the opposite figure :**

m (< C) = .....

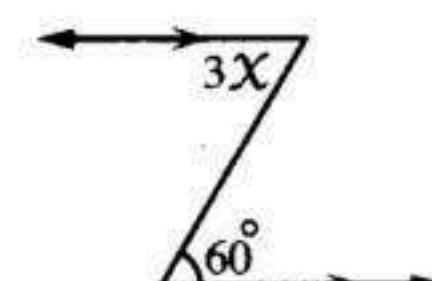
- (a)  $105^\circ$       (b)  $75^\circ$   
(c)  $45^\circ$       (d)  $90^\circ$



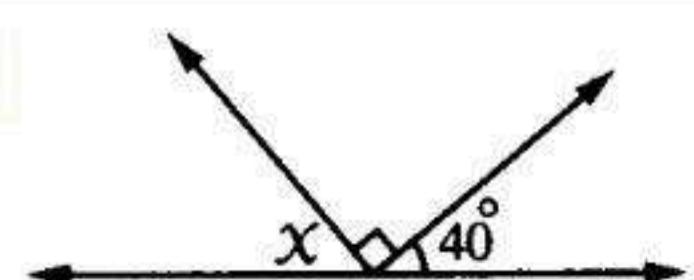
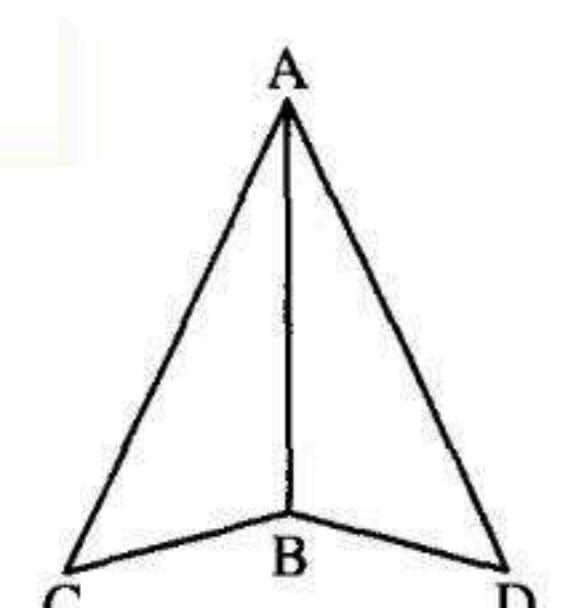
- 48

**In the opposite figure :  $x = \dots$**

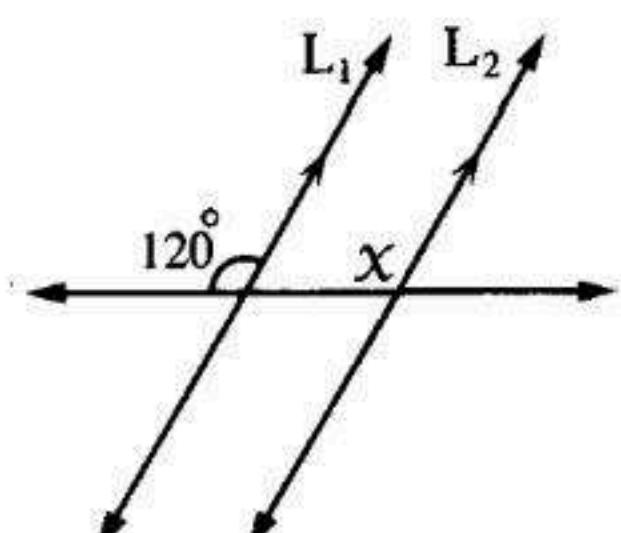
- (a) 20
  - (b) 30
  - (c) 40
  - (d) 120



## [ A ] : Complete the Following : -

- 1 The acute angle , whose measure is less than ..... and more than .....
- 2  $\overrightarrow{AB} \cup \overrightarrow{AC} = \dots$
- 3 The type of the angle of measure  $89^\circ 60'$  is .....
- 4 If the two adjacent angles are supplementary angles , then their outer sides are .....
- 5 The two bisectors of two adjacent supplementary angles are .....
- 6 The two adjacent angles formed by a straight line and a ray with a starting point on this straight line are .....
- 7 In the opposite figure :  
 $x = \dots^\circ$ 

- 8 If two straight lines intersect , then the measures of each two vertically opposite angles are .....
- 9 The two vertically opposite angles are ..... in measure.
- 10 If  $\Delta ABC \cong \Delta XYZ$  , then  $\angle CAB \equiv \angle \dots$
- 11 If the polygon ABCDE  $\cong$  the polygon XYZEF , then BC = .....
- 12 If polygon AXYD  $\cong$  polygon BXYC , then AD = .....
- 13 In the opposite figure :  
If  $\Delta ABC \cong \Delta ABD$  , the perimeter of the figure ACBD = 20 cm. ,  
AB = 6 cm. , then the perimeter of  $\Delta ABC = \dots$  cm.


- 14 The two triangles are congruent if two sides and ..... of one triangle are congruent with their corresponding parts in the other triangle.
- 15 The two right-angled triangles are congruent if .....
- 16 The two right-angled triangles are congruent if the ..... and a side of one triangle are congruent with their corresponding parts in the other triangle.
- 17 Two triangles are congruent if each ..... of one triangle is congruent to its corresponding part of the other triangle.
- 18 If  $\overleftrightarrow{AB}$  and  $\overleftrightarrow{CD}$  lie in the same plane and  $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \emptyset$ , then  $\overleftrightarrow{AB}$  and  $\overleftrightarrow{CD}$  are .....
- 19 If two straight lines are parallel to a third line , then the two straight lines are .....
- 20 If a straight line intersects two parallel straight lines , then every two corresponding angles are .....
- 21 If a straight line intersects two parallel lines , then each two alternate angles are .....
- 22 If a straight line intersects two parallel straight lines , then each two interior angles in the same side of the transversal are .....
- 23 The straight line that is perpendicular to one of two parallel lines is also .....
- 24 If two straight lines are perpendicular to a third line , then the two straight lines are .....
- 25 Two straight lines are parallel if they are cut by a transversal such that the two interior angles on one side of the transversal are .....
- 26 In the opposite figure :  
 $L_1, L_2$  are two parallel straight lines  
, then  $x = \dots$



## [ B ] : Essay Problems : -

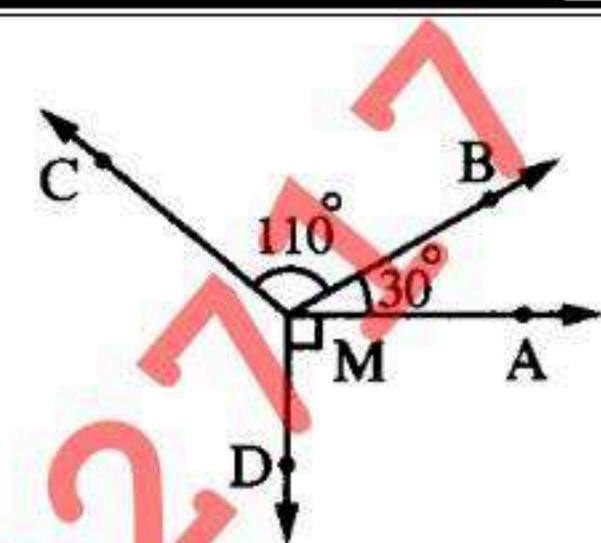
1

In the opposite figure :

$$m(\angle AMB) = 30^\circ, m(\angle BMC) = 110^\circ$$

$$, m(\angle AMD) = 90^\circ$$

Find :  $m(\angle CMD)$

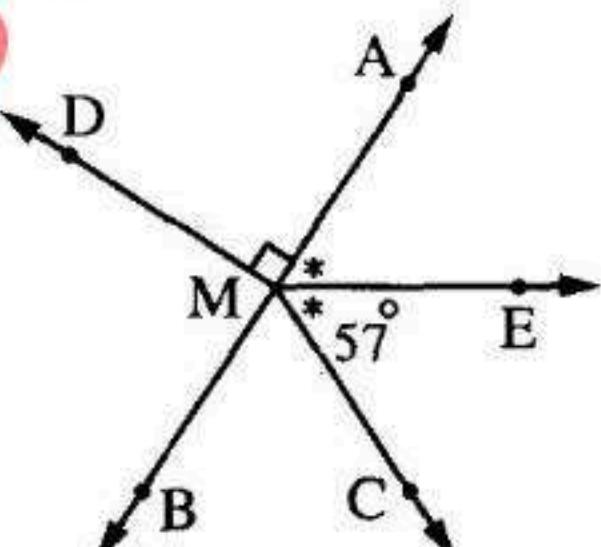


2016 Exam (15) Question (3)(b)

2

In the opposite figure :

Calculate :  $m(\angle DMC)$  (give reason)



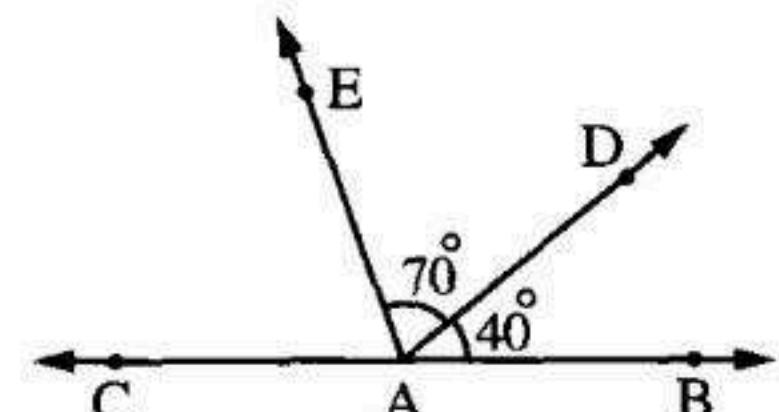
2016 Exam (2) Question (3)(b)

3

In the opposite figure :

$$m(\angle BAD) = 40^\circ, m(\angle DAE) = 70^\circ, A \in \overleftrightarrow{BC}$$

Prove that :  $\overrightarrow{AE}$  bisects  $\angle DAC$



2016 Exam (4) Question (3)(a)

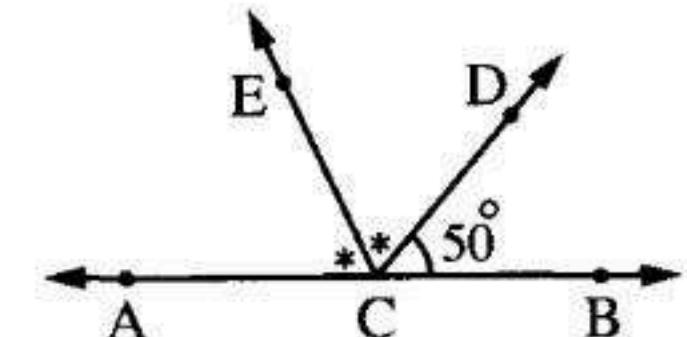
4

In the opposite figure :

$$C \in \overleftrightarrow{AB}, m(\angle BCD) = 50^\circ,$$

$\overrightarrow{CE}$  bisects  $\angle DCA$

Find :  $m(\angle ACE)$



2016 Exam (3) Question (5)(b)

5

In the opposite figure :

$$\overleftrightarrow{AC} \cap \overleftrightarrow{DE} = \{B\}, m(\angle ABD) = 50^\circ$$

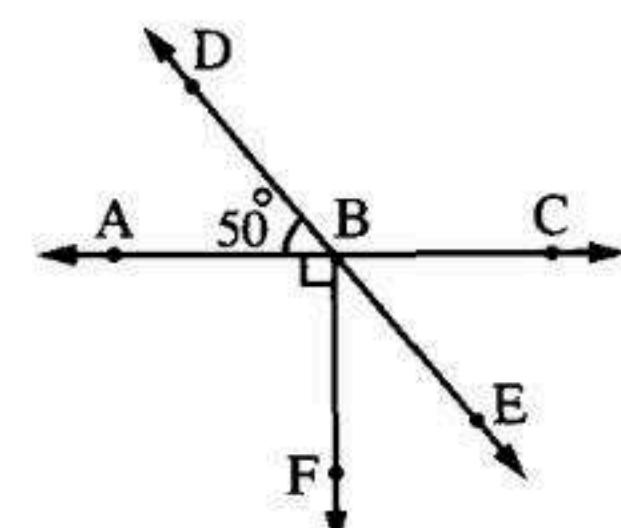
$$\text{and } m(\angle ABF) = 90^\circ$$

Find showing steps :

$$\textcircled{1} m(\angle DBC)$$

$$\textcircled{2} m(\angle CBE)$$

$$\textcircled{3} m(\angle FBE)$$



2016 Exam (12) Question (3)(a)

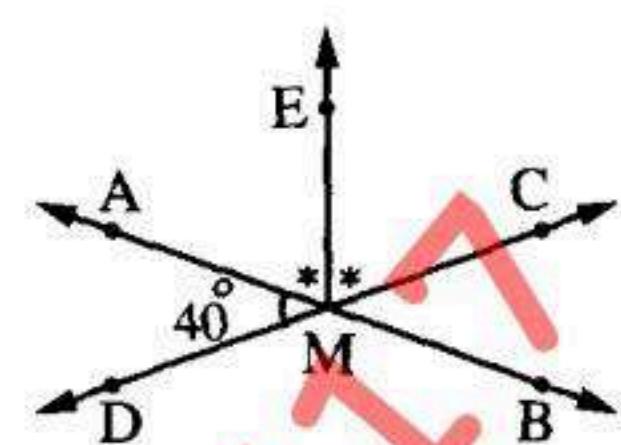
6

In the opposite figure :

$$\overleftrightarrow{AB} \cap \overleftrightarrow{DC} = \{M\}$$

,  $\overrightarrow{ME}$  bisects  $\angle AMC$  and  $m(\angle AMD) = 40^\circ$

Find :  $m(\angle AME)$ ,  $m(\angle BME)$



2016 Exam (8) Question (3)(a)

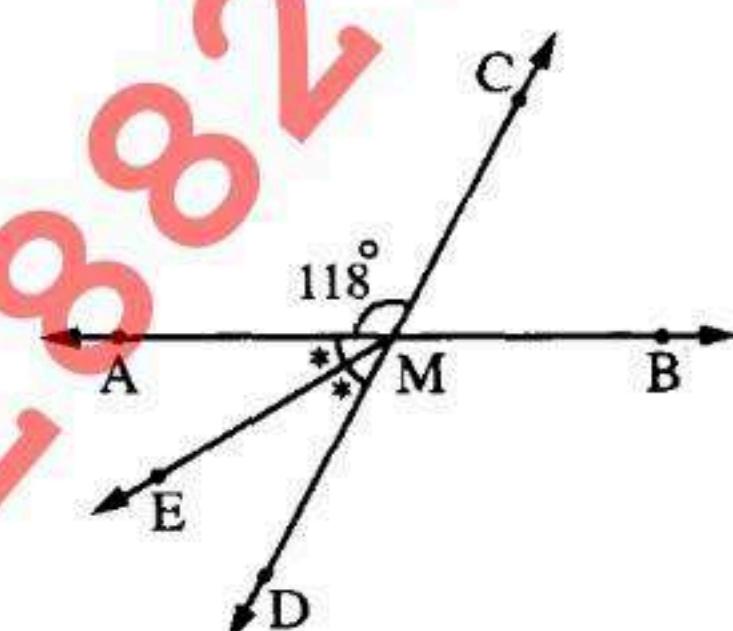
7

In the opposite figure :

$$\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{M\}, \overrightarrow{ME}$$
 bisects  $\angle AMD$

,  $m(\angle AMC) = 118^\circ$  Find :

- ①  $m(\angle BMD)$
- ②  $m(\angle BMC)$
- ③  $m(\angle AME)$



2016 Exam (6) Question (3)(a)

8

In the opposite figure :

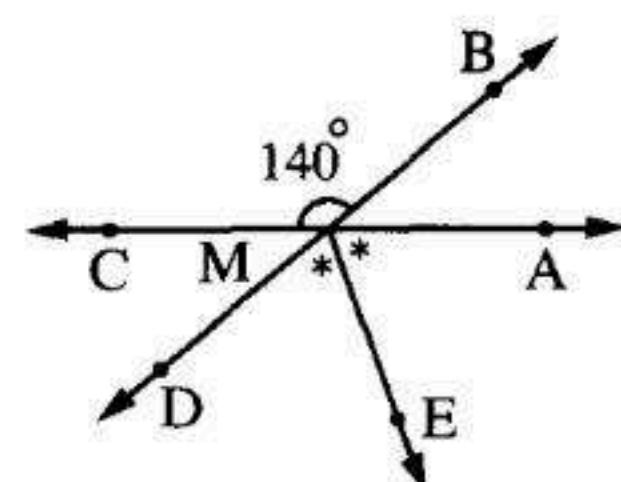
$$\overleftrightarrow{AC} \cap \overleftrightarrow{BD} = \{M\},$$

$m(\angle CMB) = 140^\circ$ ,

$\overrightarrow{ME}$  bisects  $\angle AMD$

Find : ①  $m(\angle AMB)$

②  $m(\angle AME)$



2016 Exam (1) Question (3)(b)

9

Mention two cases of congruency of two triangles.

2016 Exam (1) Question (4)(a)

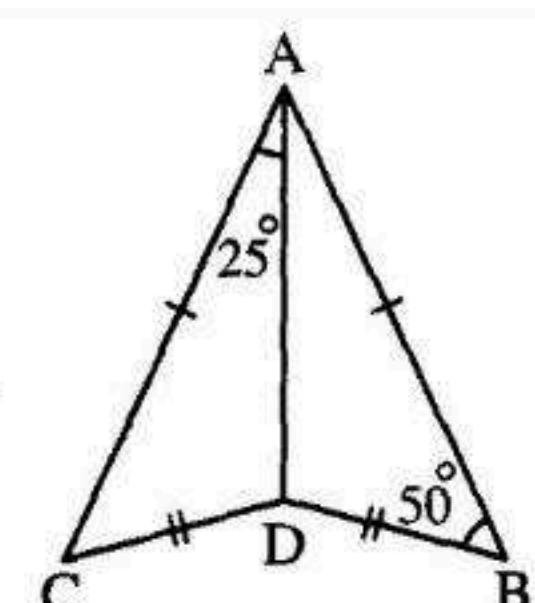
10

Complete from the opposite figure :

① The common side between the two triangles is .....

②  $\Delta ABD \cong \Delta$  .....

③  $m(\angle ADB) = \dots \circ$



2016 Exam (5) Question (4)(a)

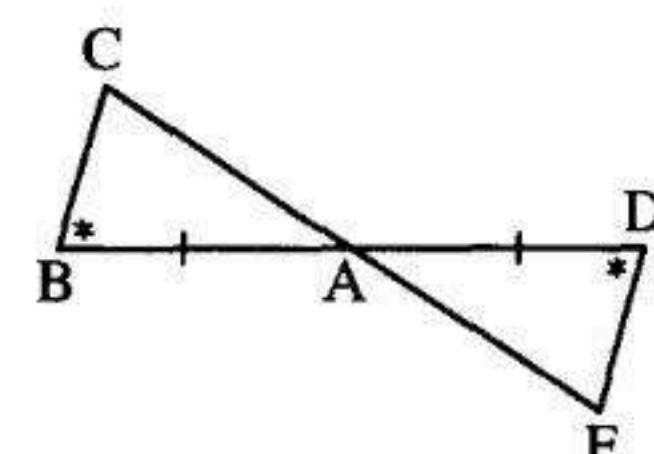
11

In the opposite figure :

$$\overline{EC} \cap \overline{BD} = \{A\}, AB = AD$$
 and

$m(\angle ABC) = m(\angle ADE)$

Does  $\Delta ADE \cong \Delta ABC$ ? Why?



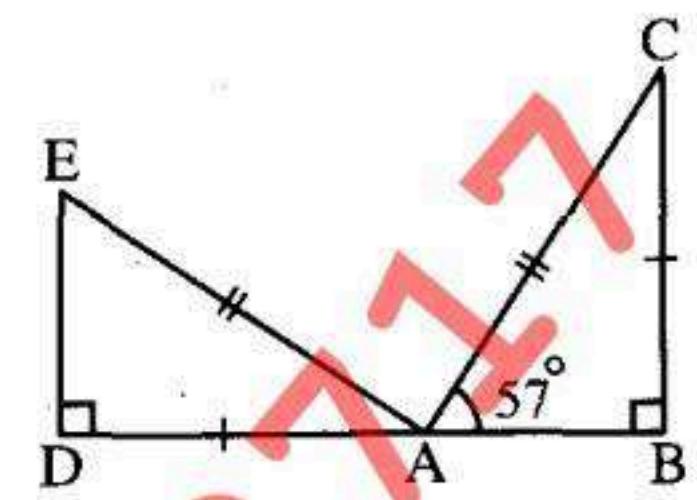
2016 Exam (13) Question (4)(b)

12

In the opposite figure :

Prove that :  $\Delta ABC \cong \Delta EDA$

, then find the measures of angles of  $\Delta ADE$



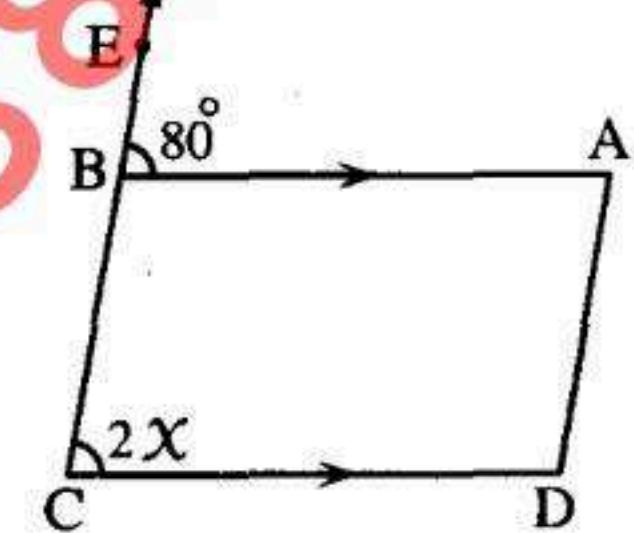
2016 Exam (3) Question (4)(a)

13

In the opposite figure :

$\overline{AB} \parallel \overline{DC}$

Find the value of :  $x$



2016 Exam (14) Question (3)(b)

14

In the opposite figure :

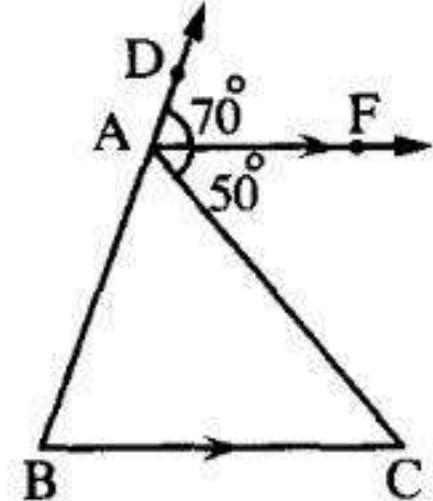
$\overrightarrow{AF} \parallel \overrightarrow{BC}$ ,  $m(\angle DAF) = 70^\circ$ ,  $m(\angle FAC) = 50^\circ$

Find :

(1)  $m(\angle B)$

(2)  $m(\angle C)$

(3)  $m(\angle BAC)$



2016 Exam (6) Question (5)(a)

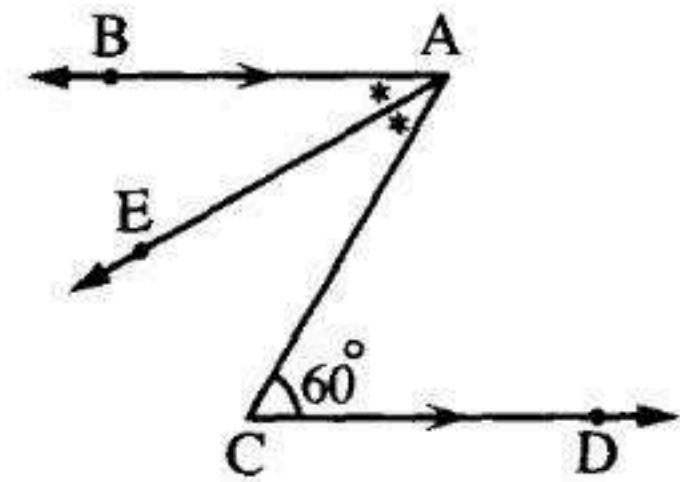
15

In the opposite figure :

$\overrightarrow{AB} \parallel \overrightarrow{CD}$ ,  $\overrightarrow{AE}$  bisects  $\angle BAC$

,  $m(\angle C) = 60^\circ$

Find :  $m(\angle BAE)$



2016 Exam (8) Question (5)(a)

16

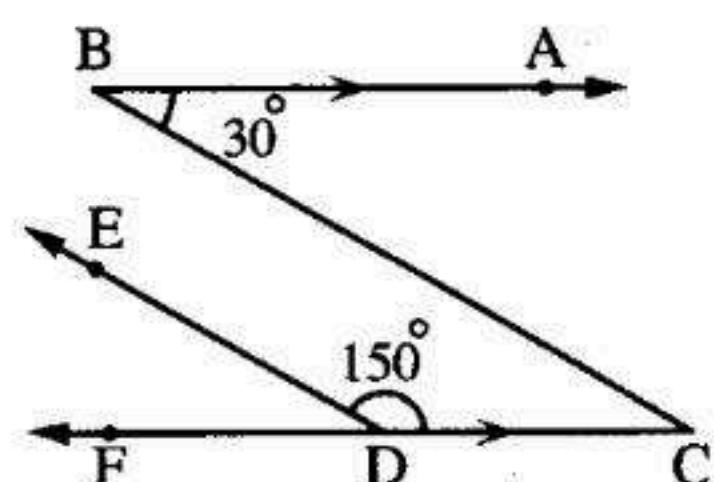
In the opposite figure :

$\overline{BA} \parallel \overline{CD}$ ,  $m(\angle ABC) = 30^\circ$

,  $m(\angle EDC) = 150^\circ$

Is  $\overline{DE} \parallel \overline{CB}$ ? Why?

(Write the steps of your answer)



2016 Exam (9) Question (5)(b)

17

Draw  $\angle ABC$  whose measure is  $60^\circ$  and bisect it. (Don't remove the arcs)

2016 Exam (12) Question (4)(a)

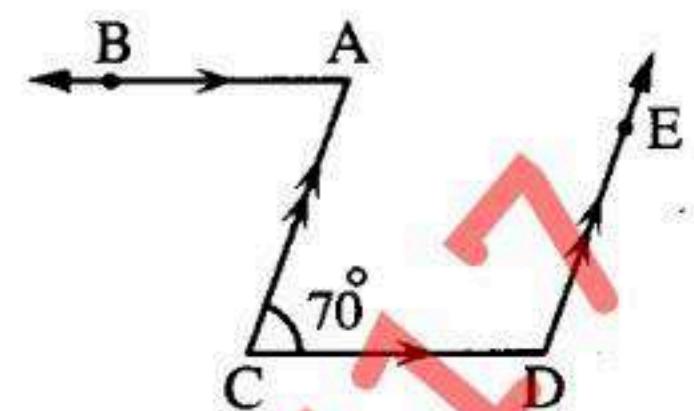
18

In the opposite figure :

$$\overrightarrow{AB} \parallel \overrightarrow{DC}, \overrightarrow{CA} \parallel \overrightarrow{DE}, m(\angle C) = 70^\circ$$

Find : (1)  $m(\angle A)$  (give reason)

(2)  $m(\angle D)$  (give reason)



2016 Exam (1) Question (5)(b)

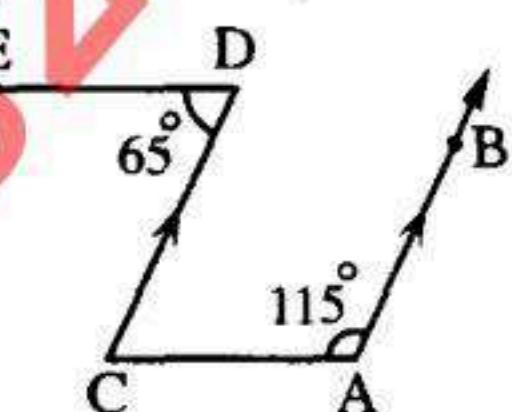
19

In the opposite figure :

$$\overrightarrow{AB} \parallel \overrightarrow{CD}, m(\angle BAC) = 115^\circ,$$

$$m(\angle EDC) = 65^\circ$$

Does  $\overrightarrow{AC} \parallel \overrightarrow{DE}$ ? Why?



2016 Exam (13) Question (5)(a)

20

In the opposite figure :

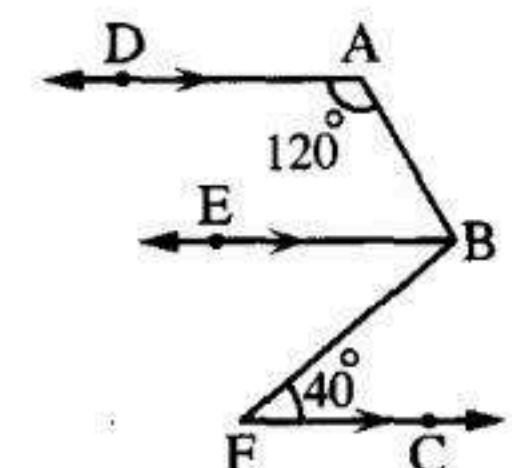
$$\overrightarrow{AD} \parallel \overrightarrow{BE} \parallel \overrightarrow{FC}, m(\angle A) = 120^\circ$$

$$, m(\angle BFC) = 40^\circ$$

Find : (1)  $m(\angle ABE)$

(2)  $m(\angle FBE)$

(3)  $m(\angle ABF)$



2016 Exam (6) Question (4)(b)

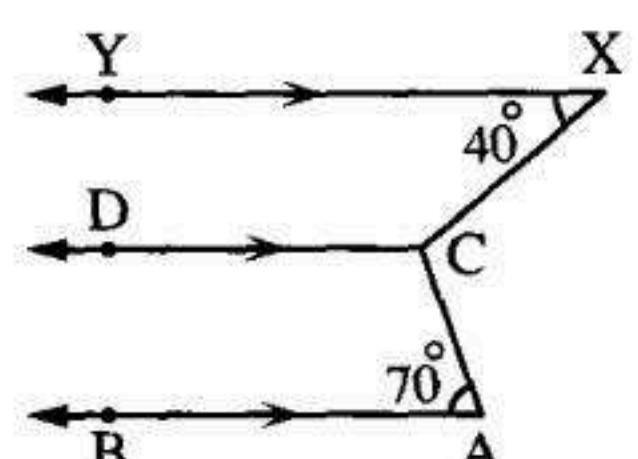
21

In the opposite figure :

$$\overrightarrow{AB} \parallel \overrightarrow{CD} \parallel \overrightarrow{XY},$$

$$m(\angle A) = 70^\circ, m(\angle X) = 40^\circ$$

Find :  $m(\angle ACX)$  and  $m(\angle DCX)$



2016 Exam (7) Question (3)(b)

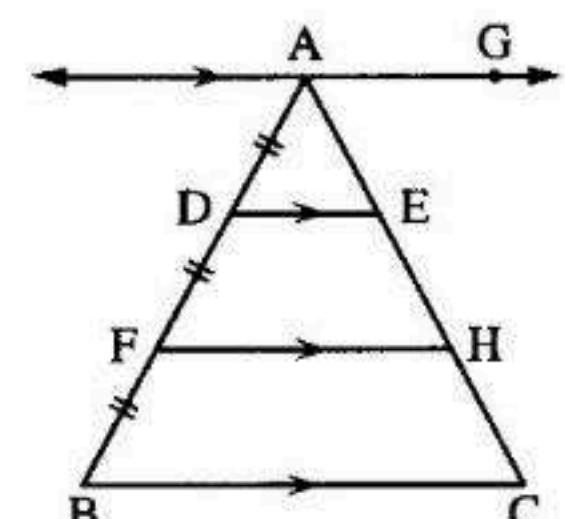
22

In the opposite figure :

$$\overrightarrow{AG} \parallel \overrightarrow{DE} \parallel \overrightarrow{FH} \parallel \overrightarrow{BC} \text{ and } AC = 12 \text{ cm.}$$

If  $AD = DF = FB$

, find the length of :  $\overline{AH}$



2016 Exam (11) Question (5)(b)

23

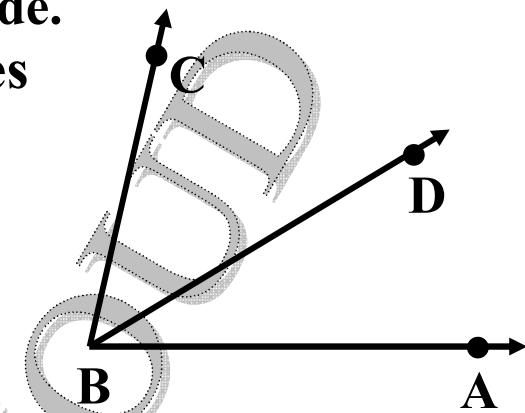
Draw  $\overline{AB}$  where  $AB = 5 \text{ cm.}$ , using ruler and compasses draw the axis of symmetry of  $\overline{AB}$  (Do not remove the arcs).

2016 Exam (11) Question (3)(b)

## Adjacent angles:

Two angles are said to be adjacent if they have a common vertex, a common side and the other two sides on opposite sides of this common side.

$\angle ABD, \angle DBC$  are adjacent angles



## Complementary angles:

Two angles are said to be complementary if their sum is  $90^\circ$

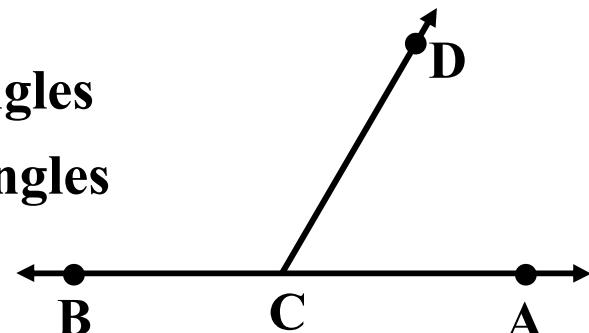
## Supplementary angles:

Two angles are said to be supplementary if their sum is  $180^\circ$

Remark: Two adjacent angles formed by a straight line and a ray with a starting point on this straight line are supplementary

$\angle ACD, \angle DCB$  are adjacent angles

And they are supplementary angles

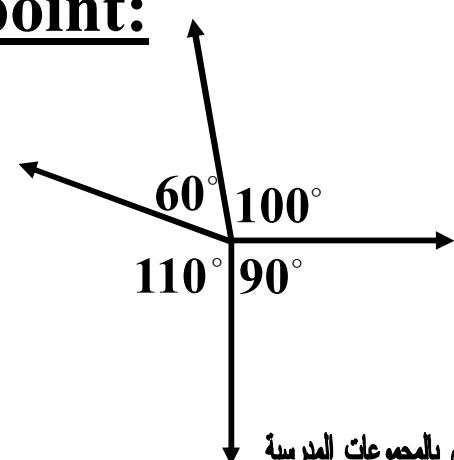


## Accumulative angles at a point:

The sum of measures of the

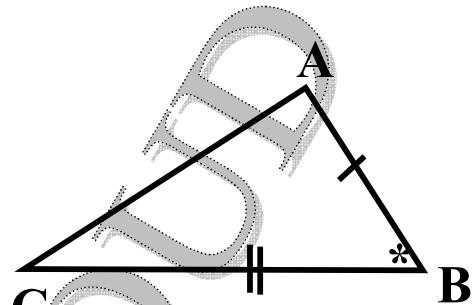
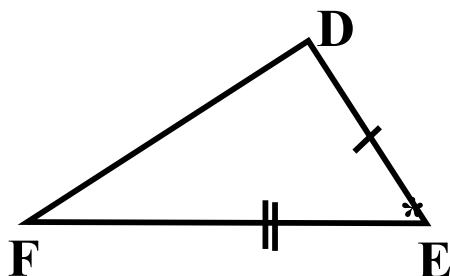
Accumulative angles

At a point is  $360^\circ$



## The first case for congruency (S.A.S)

Two triangles are congruent if two sides and the included angle of one are congruent with the corresponding parts of the other



If in the two triangles , ABC and DEF

$$\left\{ \begin{array}{l} \overline{AB} \equiv \overline{DE} \\ \overline{BC} \equiv \overline{EF} \\ \angle B \equiv \angle E \end{array} \right.$$

Then  $\Delta ABC \equiv \Delta DEF$  and from congruency, we get

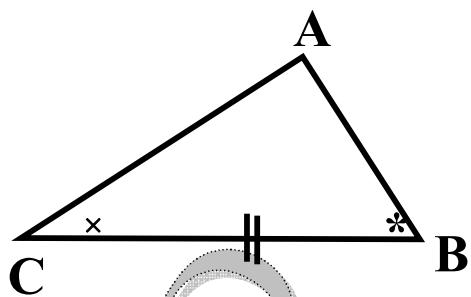
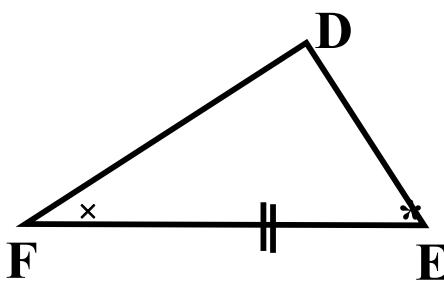
$$\overline{AC} \equiv \overline{DF}, \angle A \equiv \angle D \text{ and } \angle C \equiv \angle F$$

## The second case for congruency (S.A.A)

If the two angles and the side drawn between their vertices of one of the two triangles are congruent to the corresponding parts of the other triangle, then the two triangles are congruent.

If in the two triangles , ABC and DEF

$$\left\{ \begin{array}{l} \overline{BC} \equiv \overline{EF} \\ \angle B \equiv \angle E \\ \angle C \equiv \angle F \end{array} \right.$$

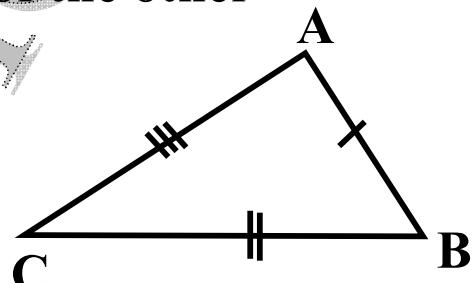
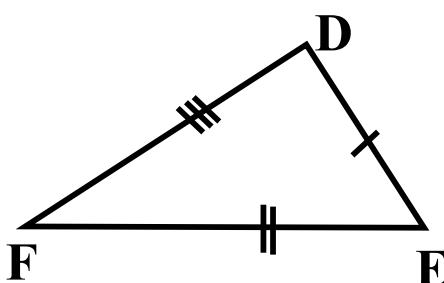


Then  $\Delta ABC \equiv \Delta DEF$  and from congruency, we get

$\overline{AB} \equiv \overline{DE}$ ,  $\overline{AC} \equiv \overline{DF}$  and  $\angle A \equiv \angle D$

### The third case for congruency (S.S.S)

Two triangles are congruent if each side of one triangle is congruent to its corresponding side of the other triangle.



If in the two triangles , ABC and DEF

$$\left. \begin{array}{l} \overline{AB} \equiv \overline{DE} \\ \overline{AC} \equiv \overline{DF} \\ \overline{BC} \equiv \overline{EF} \end{array} \right\}$$

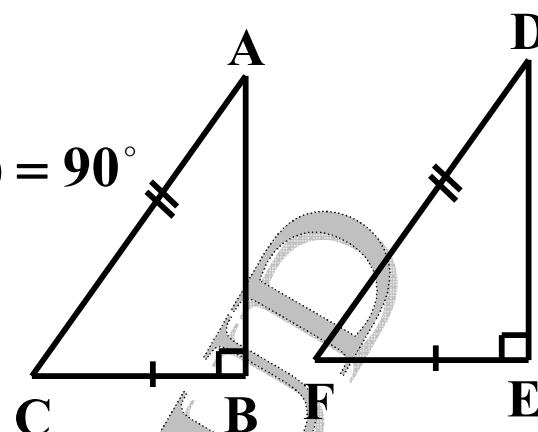
Then  $\Delta ABC \equiv \Delta DEF$  and from congruency, we get  
 $\angle A \equiv \angle D, \angle B \equiv \angle E$  and  $\angle C \equiv \angle F$

### The fourth case for congruency.

Two right-angled triangles are congruent, if the hypotenuse and one side of one triangle are congruent to their corresponding parts of the other triangle.

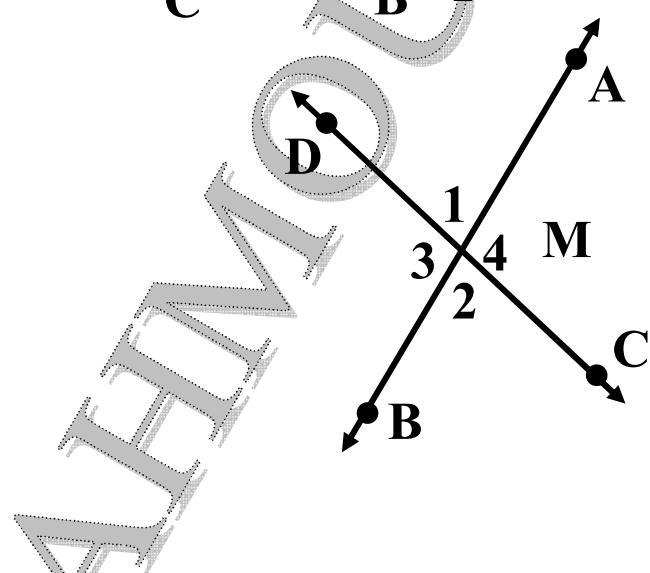
If in the two triangles , ABC and DEF

$$\left\{ \begin{array}{l} \overline{AC} \equiv \overline{DF} \\ \overline{BC} \equiv \overline{EF} \\ m(\angle B) = m(\angle E) = 90^\circ \end{array} \right.$$

V.O.A.

$$m(\angle 1) = m(\angle 2)$$

$$m(\angle 3) = m(\angle 4)$$



Then  $\Delta ABC \cong \Delta DEF$  and from congruency, we get

$$\overline{AB} \equiv \overline{DE}, \angle A \equiv \angle D \text{ and } \angle C \equiv \angle F$$

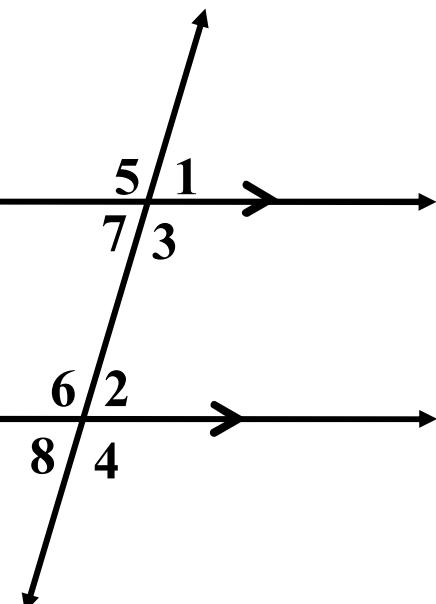
Parallel straight lines:Corresponding angles:

$$m(\angle 1) = m(\angle 2), m(\angle 3) = m(\angle 4)$$

$$m(\angle 5) = m(\angle 6), m(\angle 7) = m(\angle 8)$$

Alternate angles:

$$m(\angle 3) = m(\angle 6), m(\angle 2) = m(\angle 7)$$

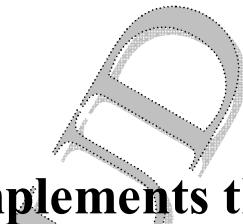
Interior supplementary angles:Angles:

$$m(\angle 3) + m(\angle 2) = 180^\circ, \quad m(\angle 7) + m(\angle 6) = 180^\circ$$

**[1] Complete:**

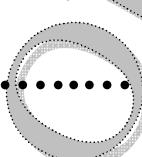
**1) The sum of measures of the accumulative angles at a point = .....**

.....



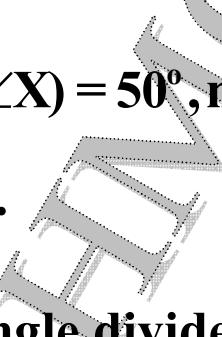
**2) The angle whose measure is  $72^\circ$  complements the angle whose measure is .....**

.....



**3) If  $\Delta ABC \cong \Delta XYZ$  and  $m(\angle X) = 50^\circ$ ,  $m(\angle B) = 60^\circ$  then  $m(\angle Z) = .....$**

.....



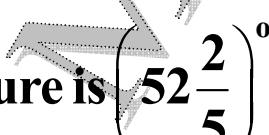
**4) The diagonal of the rectangle divides its surface into two ..... triangles.**

.....



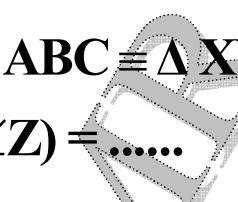
**5) The angle of measure is  $\left(52\frac{2}{5}\right)^\circ$  is supplemented by the angle of measure .....**

.....



**6) If  $\Delta ABC \cong \Delta XYZ$  and  $m(\angle A) + m(\angle B) = 130^\circ$ , then  $m(\angle Z) = .....$**

.....



**7) If  $m(\angle A) = 150^\circ$ , then  $m(\text{reflex } \angle A) = .....$**

8) The two adjacent complementary angles , their

terminal sides are .....

9) If a line segment is extended from one side without

limit, the produced figure is .....

10) If  $\angle A$  supplements  $\angle B$ ,  $\angle A \equiv \angle B$ , then  $m(\angle B) = \dots$

11) The measure of the straight angle = .....

12) In the right-angled triangle , the area of the square

set up the hypotenuse equals .....

13) If one of the two supplement angles is acute then the

other is ..... angle.

14) The two triangles are congruent if two sides and

..... in one of them are

congruent to their corresponding elements in the other.

15) If  $m(\angle A) = 170^\circ$ , then  $m(\text{reflex } \angle A) = \dots$

16) ..... < the measure of the obtuse angle < .....

17) If  $\triangle XYZ$  is right-angled at  $X$ ,  $XY = 12\text{cm}$ ,  $XZ = 9\text{cm}$ .  
then  $(YZ)^2 = \dots \text{cm}^2$ .

18) The required condition for the two straight lines are parallel is .....

19) If  $\triangle ABC \cong \triangle XYZ$  then  $BC = \dots$

20) If  $\angle A$  supplements  $\angle B$ , and  $m(\angle A) = 2 m(\angle B)$ ,  
then  $m(\angle B) = \dots$

21) The angle of measure is  $89^\circ 60'$  is ..... angle.

22) The number of edges which are parallel to one edge of a cube is .....

23) If  $\overline{AB} \equiv \overline{XY}$ , then  $AB - XY = \dots$

24) The sum of measures of the two complementary

angles = .....

25) The angle of measure is  $x^\circ$  complements the angle of  
measure is .....

26) If ABC is a triangle in which  $AB = 5\text{ cm}$ ,  $BC = 12\text{ cm}$   
and  $AC = 13\text{ cm}$  then  $m(\angle \dots) = 90^\circ$

27) The two right-angled triangles are congruent if ....  
in one of them are congruent with their  
corresponding elements in the other triangle.

28) The angle whose measure  $47^\circ 30'$  is complemented  
the angle whose measure .....

29) The sum of measures of the two supplementary  
angles equals .....

30) If  $m(\angle X) = \frac{1}{2}m(\angle Y)$  and  $m(\angle X) = 30^\circ$ , then the two  
angles X and Y are.....

31) In the right-angled triangle , the area of the square

set up the hypotenuse equals .....

32) The two straight lines which are parallel to a third

straight line are .....

33) If  $m(\angle X) = 2m(\angle Y)$  and Y is obtuse angle, then  $\angle X$  is ....

34) If  $\Delta ABC \cong \Delta XYZ$  and  $m(\angle X) + m(\angle Z) = 140^\circ$ ,  
then  $m(\angle B) = ....$

35) The two adjacent angles formed by intersecting a

straight line and a ray whose start point lies on the  
straight line are .....

36) In  $\Delta ABC$ : if  $(AB)^2 - (BC)^2 = (AC)^2$ ,  
then  $m(\angle....) = 90^\circ$

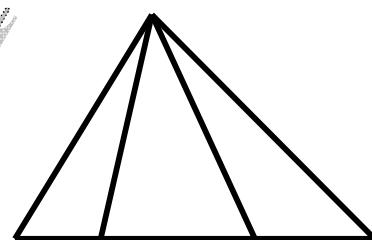
37) If  $m(\angle A) = 50^\circ$ ,  $\angle A$  complements  $\angle B$ ,  $\angle B$  supplements  
 $\angle C$ , then  $m(\angle C) = ....$

38) A rectangle of length 4 cm. and width 3 cm , then the  
area of the square set its diagonal equals .....  $\text{cm}^2$

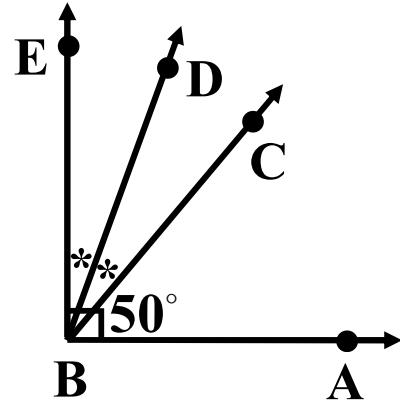
39) If the lengths of the two sides of the right angle in the right-angled triangle are 5 cm. and 6 cm. then the area of the square set up the hypotenuse = ..... cm<sup>2</sup>

40) If  $m(\angle X) = \frac{1}{2} m(\angle Y)$  and  $m(\angle X) = 60^\circ$ , then the two angles X and Y are .....

41) The number of triangles in the opposite figure is .....



42) In the opposite figure:

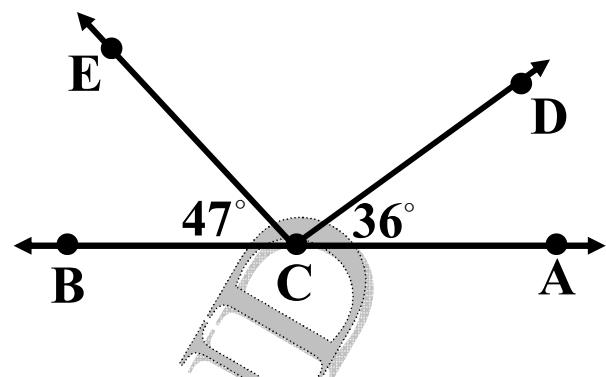


If  $m(\angle ABC) = 50^\circ$ ,  $BD$  bisects  $\angle CBE$

$\overrightarrow{BD} \perp \overrightarrow{BE}$ , then  $m(\angle CBD) = .....$

43) In the opposite figure:

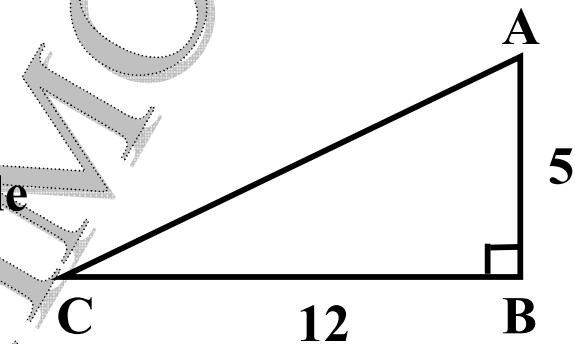
$$m(\angle DCE) = \dots\dots$$



44) In the opposite figure:

ABC is a right-angled triangle

$$\text{at } B \text{ then } (AC)^2 = \dots \text{ cm}^2.$$



45) In the opposite figure:

If  $\triangle ABC$  is

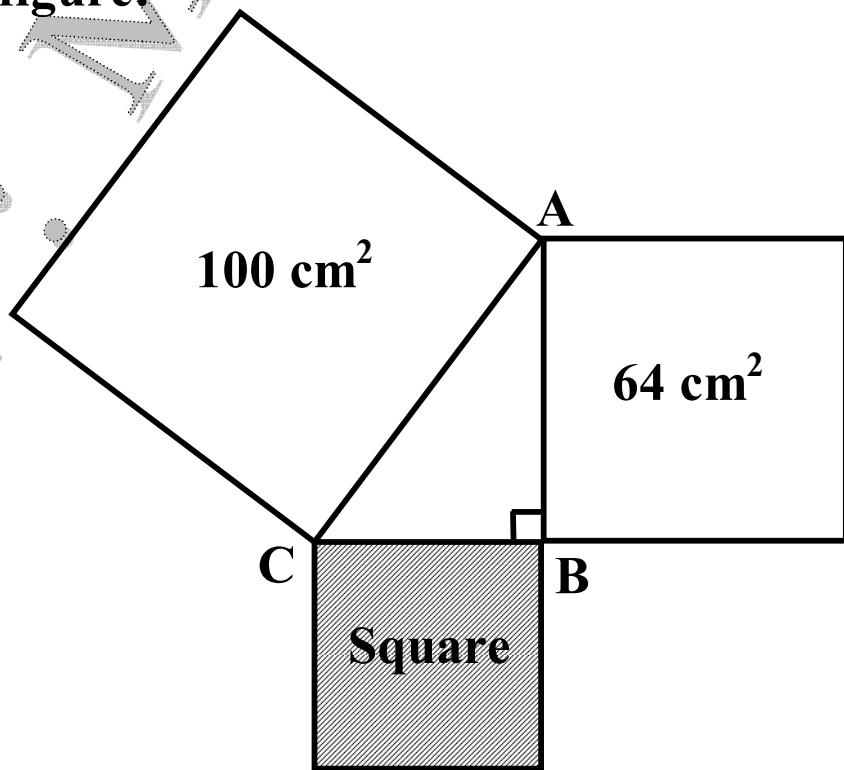
right-angled

at B then the

area of the

shaded square

equals .....  $\text{cm}^2$ .



46) In the opposite figure:

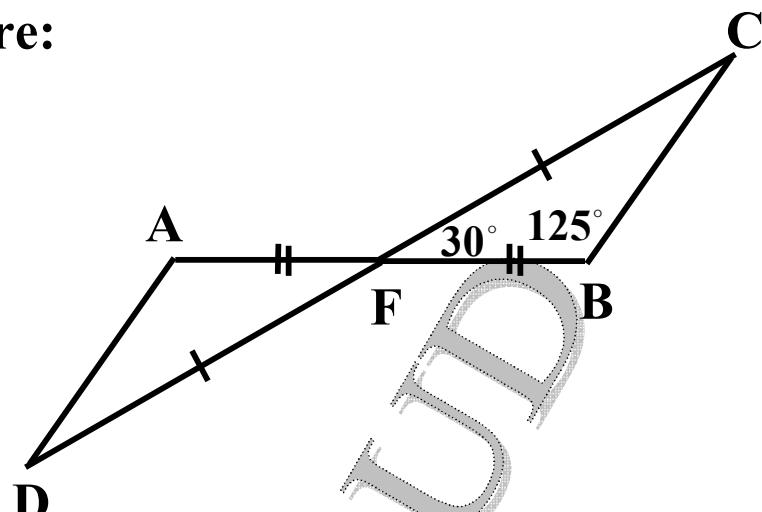
If  $\overline{AB} \cap \overline{CD} = \{F\}$

$FC = FD, FA = FB$

$m(\angle CFB) = 30^\circ$

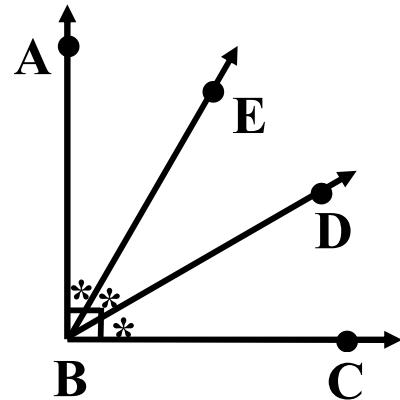
and  $m(\angle B) = 125^\circ$

then  $m(\angle D) = \dots$



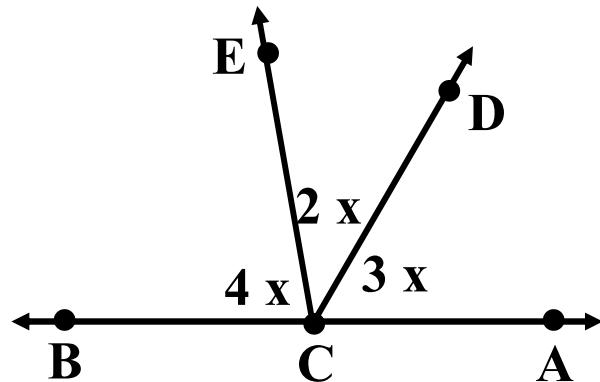
47) In the opposite figure:

If  $\overrightarrow{BA} \perp \overrightarrow{BC}$ , then  $m(\angle CBE) = \dots$



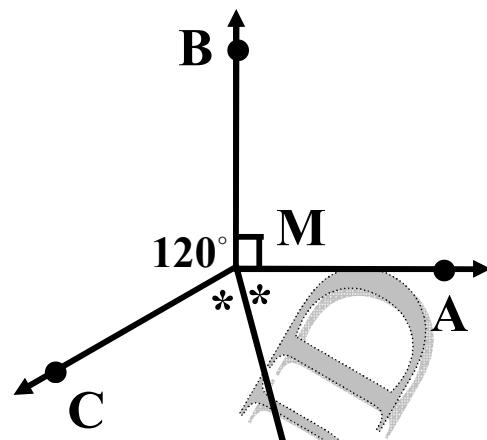
48) In the opposite figure:

If  $A \in \overleftrightarrow{BC}$ , then  $x = \dots$



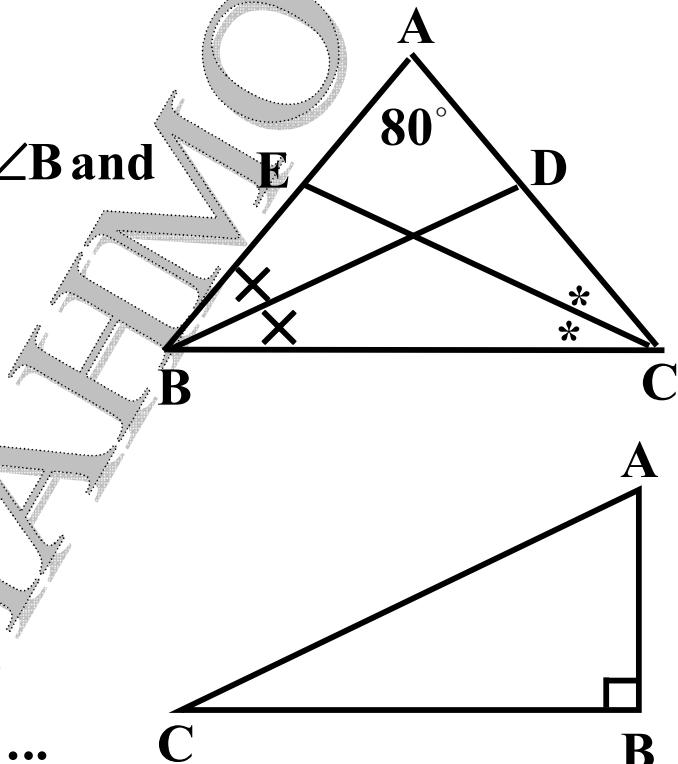
49) In the opposite figure:

$\overrightarrow{MD}$  bisects  $\angle AMC$ ,  
then  $m(\angle AMD) = \dots\dots$

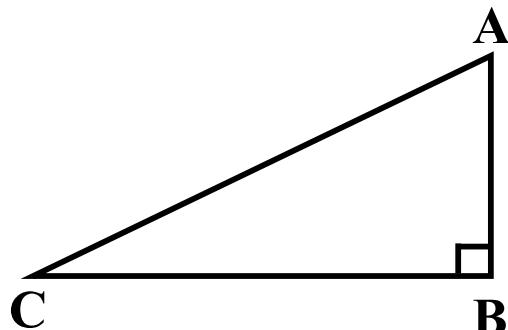


50) In the opposite figure:

$m(\angle A) = 80^\circ$ ,  $\overrightarrow{BD}$  bisects  $\angle B$  and  
 $\overrightarrow{CE}$  bisects  $\angle C$ ,  
then  $m(\angle CFB) = \dots\dots$

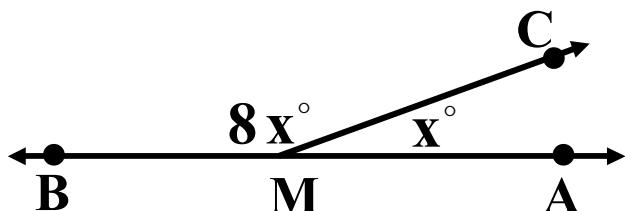


51) The hypotenuse in the  
opposite triangle is .....  
.....



52) In the opposite figure:

If  $M \in AB$ , then  $x = \dots\dots$



53) If  $\triangle XYZ$  is

right- angled

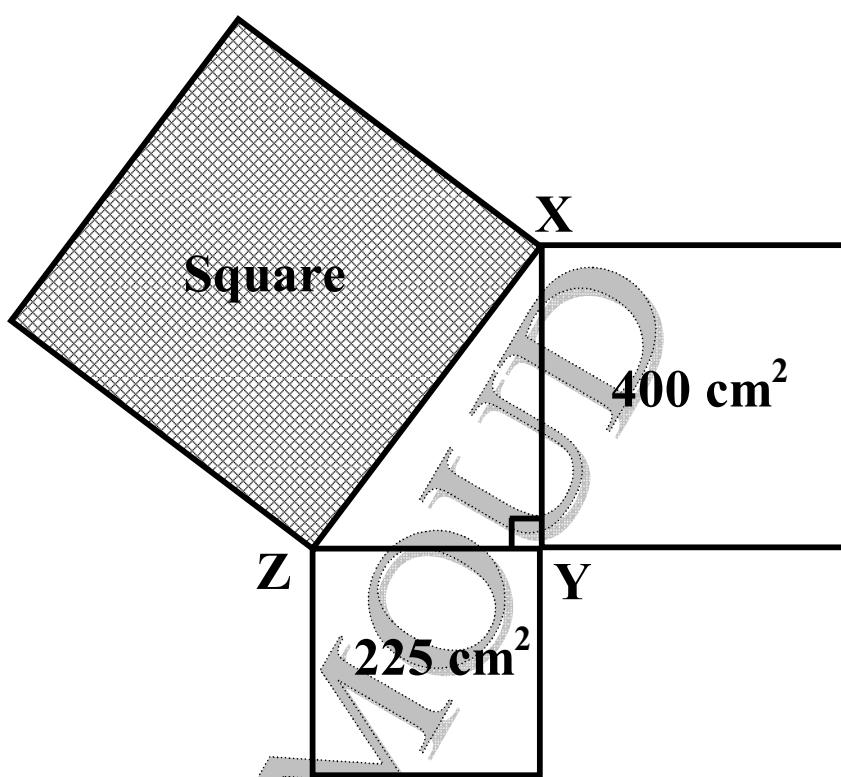
triangle at

$Y$ , then

the area of

shaded square

$= \dots \text{cm}^2$



54) In the opposite figure:

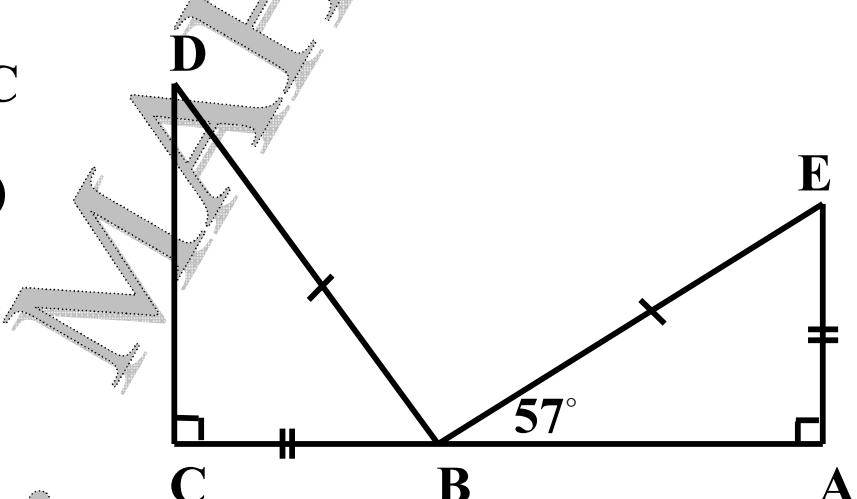
$B \in \overleftrightarrow{AC}$ ,  $AE = BC$

$BE = BD$ ,  $m(\angle A)$

$= m(\angle C) = 90^\circ$ ,

$m(\angle EBA) = 57^\circ$

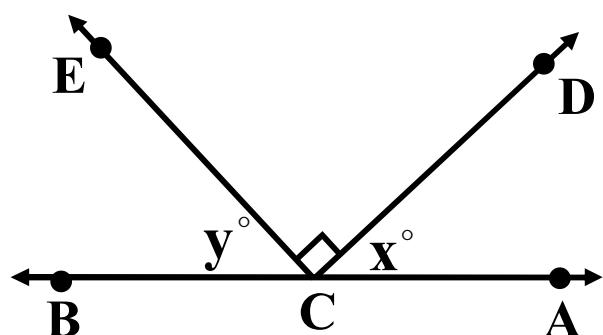
then  $m(\angle EBD) = \dots \dots \dots$



55) In the opposite figure:

If  $C \in \overleftrightarrow{AB}$

then  $x^\circ + y^\circ = \dots \dots \dots$

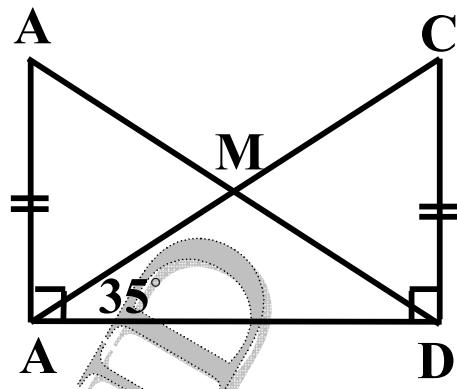


56) In the opposite figure:

If  $AB = CD$ ,  $m(\angle CBD) = 35^\circ$

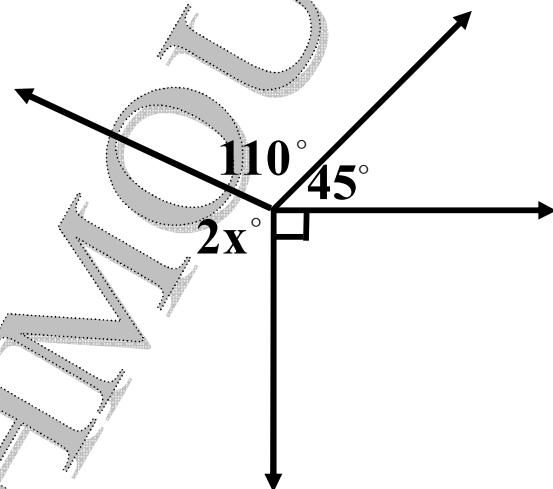
$\overline{AB} \perp \overline{DB}$  and  $\overline{CD} \perp \overline{DB}$

then  $m(\angle DMB) = \dots\dots$



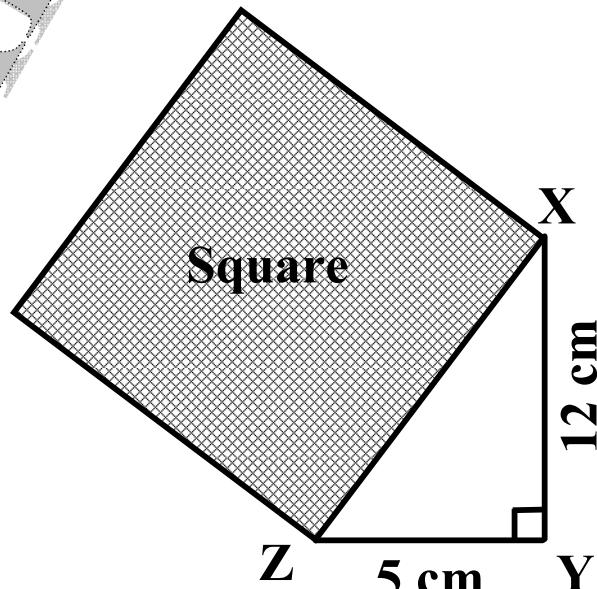
57) In the opposite figure:

$x = \dots$



58) In the opposite figure:

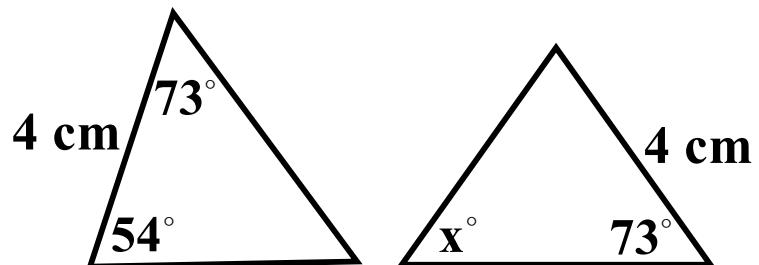
The area of the shaded square = .....  $\text{cm}^2$ .



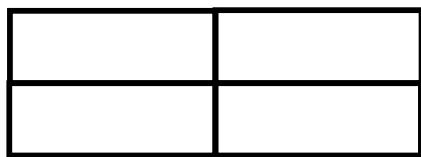
59) In the opposite figure:

If the two triangles  
are congruent

then  $x = \dots$



60) The number of rectangles



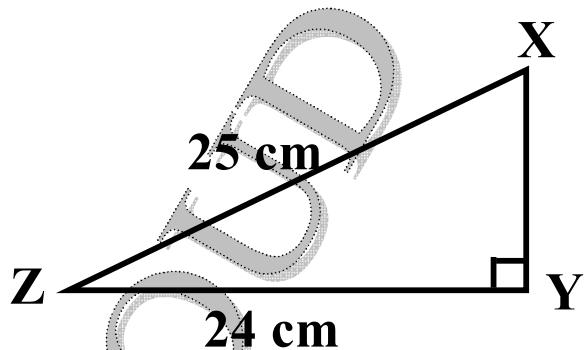
in the opposite figure = .....

61) In the opposite figure:

If  $XYZ$  is a right-angled

triangle at  $Y$

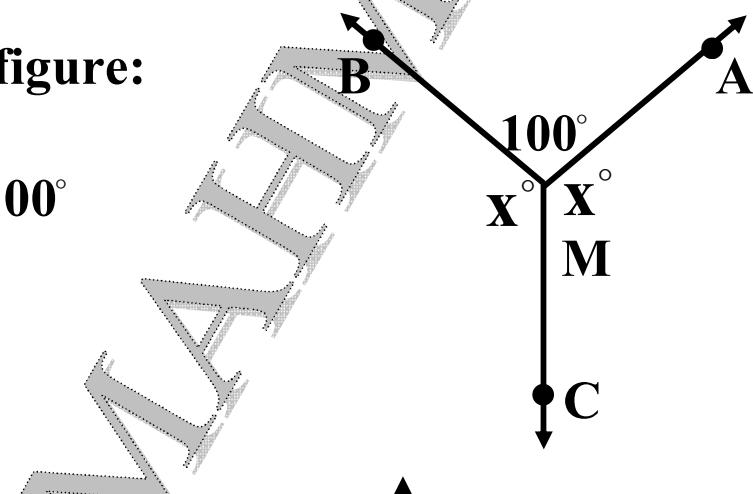
then  $(XY)^2 = \dots \text{ cm}^2$ .



62) In the opposite figure:

If  $m(\angle AMB)=100^\circ$

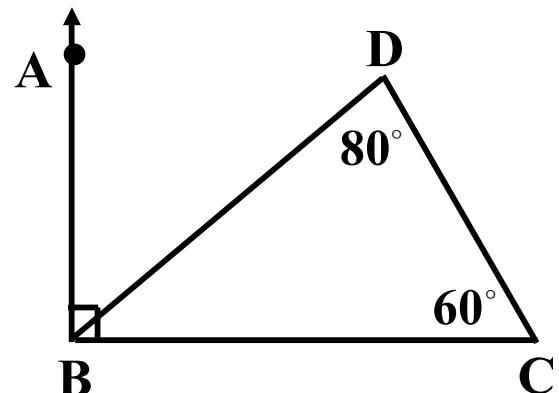
, then  $x = \dots$



63) In the opposite figure:

If  $\overrightarrow{BA} \perp \overrightarrow{BC}$ , then

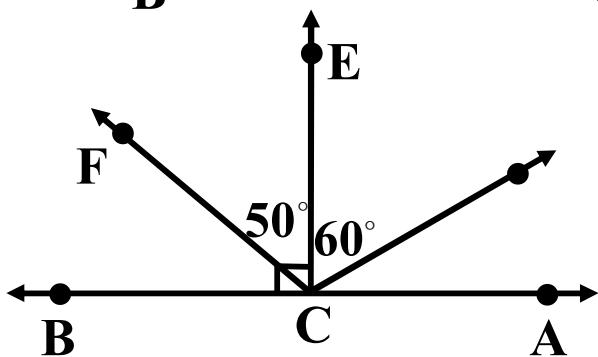
$m(\angle ABD) = \dots$



64) The number of obtuse

angle in the opposite

Figure is .....



65) In the opposite figure:

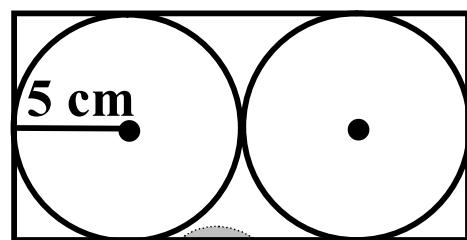
Two circles are drawn inside

a rectangle M and N each of

them is of radius 5 cm long ,

then the area of

$$\text{rectangle} = \dots \text{cm}^2 .$$



66) In the opposite figure:

If  $\triangle ABC$  is

right-angled at

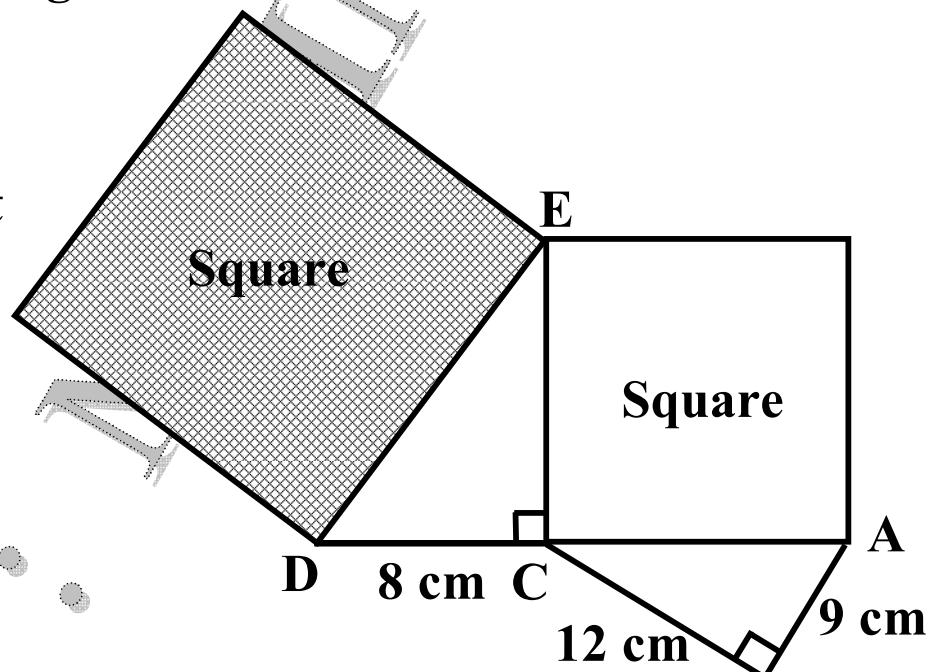
B and  $\triangle ECD$  is

Right-angled

at C, then

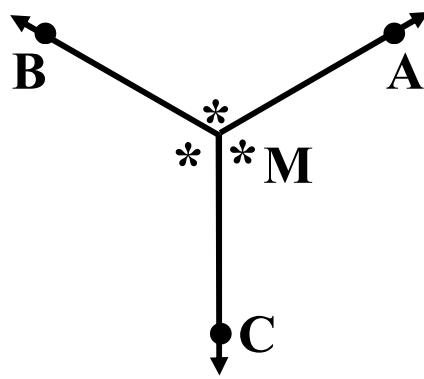
the area of the

$$\text{shaded square} = \dots \text{cm}^2 .$$



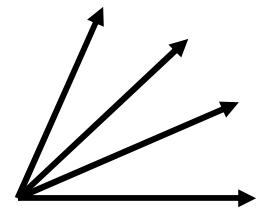
67) In the opposite figure:

$$m(\angle AMC) = \dots \text{.}$$



68) In the opposite figure:

The number of the acute angles

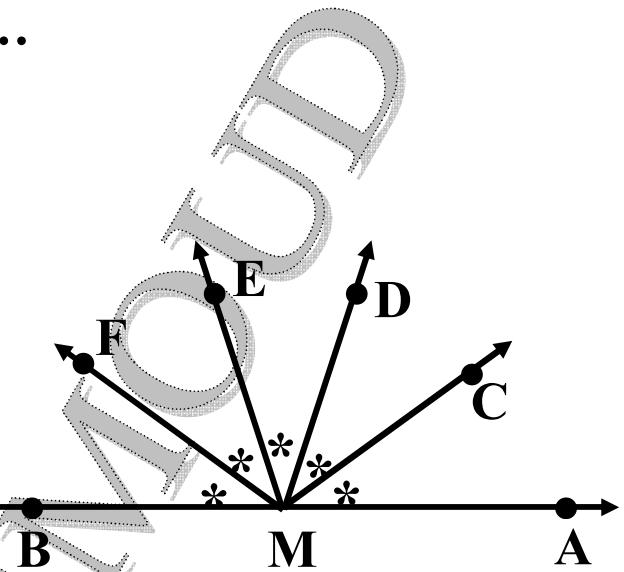


in the opposite figure is .....

69) In the opposite figure:

If  $M \in \overleftrightarrow{AB}$ , then

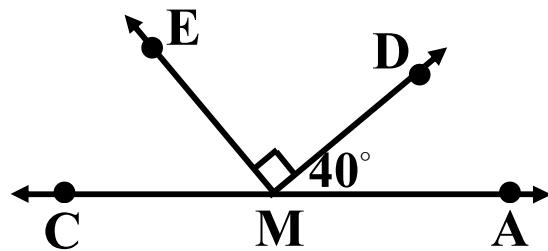
$m(\angle AMC) = \dots\dots$



70) In the opposite figure:

If  $M \in \overleftrightarrow{AC}$ , then

$m(\angle EMC) = \dots\dots$



71) In the opposite figure:

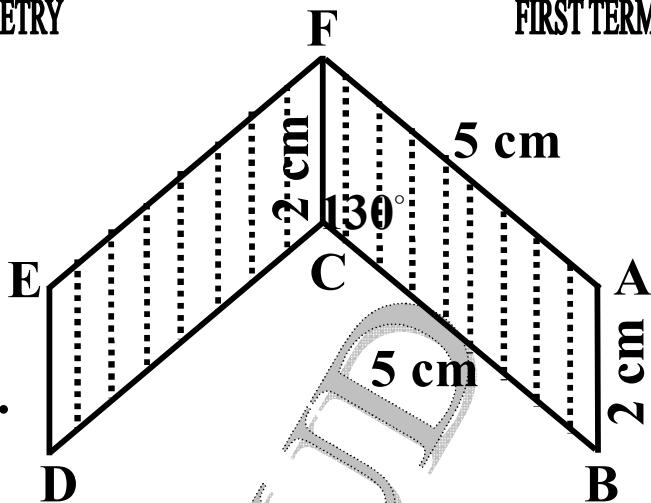
If the figure ABCF =

the figure EDCF, then

a) The perimeter of the

shaded figure = ..... cm.

b)  $m(\angle BCD) = \dots$



72) In the opposite figure:

If  $\triangle CDE$  is right-angled at C,  $CD = 4 \text{ cm}$

$CE = 3 \text{ cm}$ .

Complete

the following

a) The area of the

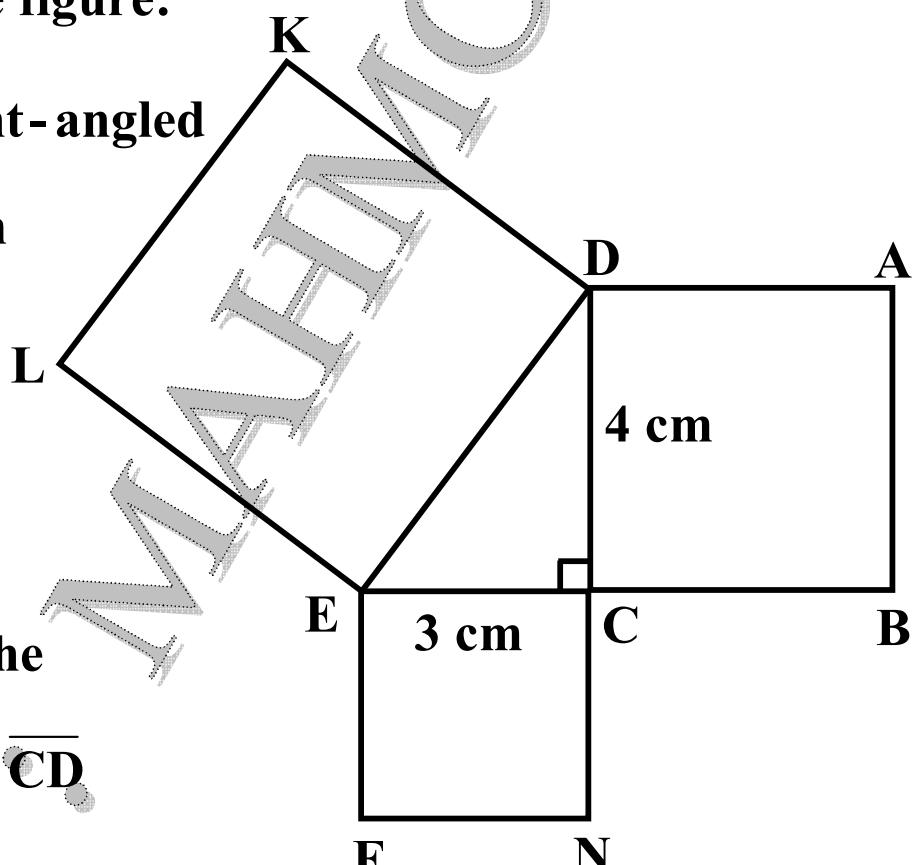
squares set up  $\overline{CD}$

= .....  $\text{cm}^2$ .

b) The area of the square

set up  $\overline{DE} = ..... \text{cm}^2$ .

c) The area of all figure = .....  $\text{cm}^2$ .



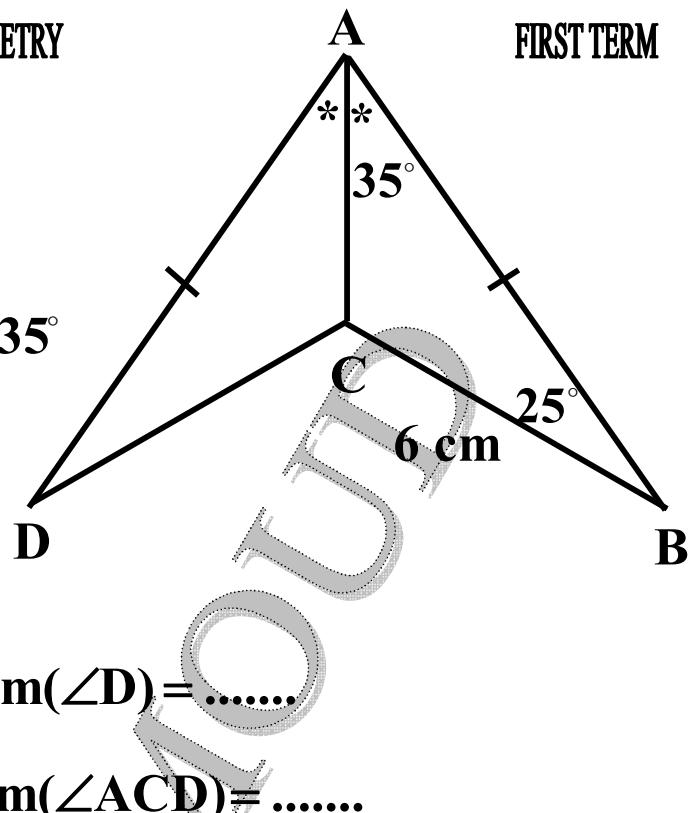
73) In the opposite figure:

If  $AB = AD$ ,  $BC = 6 \text{ cm}$ .

$$m(\angle BAC) = m(\angle DAC) = 35^\circ$$

$m(\angle B) = 25^\circ$  Complete

the following:



a)  $\triangle ACB \cong \dots$

b)  $m(\angle D) = \dots$

c)  $CD = \dots \text{ cm}$

d)  $m(\angle ACD) = \dots$

74) In the opposite figure:

If  $D \in \overleftrightarrow{CE}$ ,  $\overline{BC} \perp \overline{CD}$

the figure  $ABCD \cong$  the

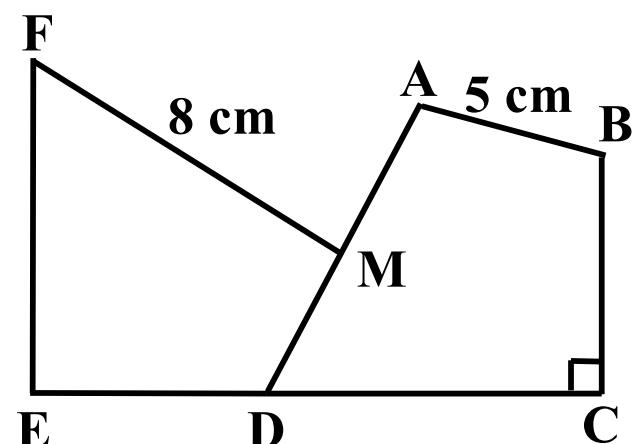
figure MDEF complete

the following:

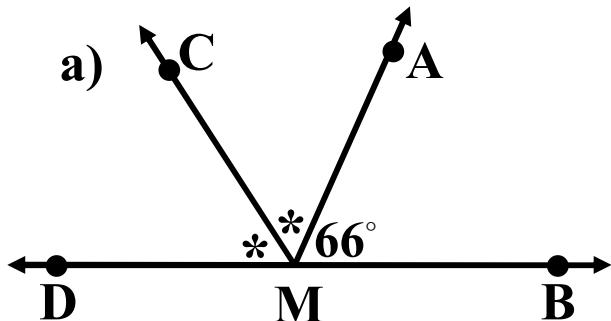
a)  $m(\angle B) = m(\angle \dots)$

b)  $m(\angle B) + m(\angle F) = \dots$

c)  $AM = \dots \text{ cm}$



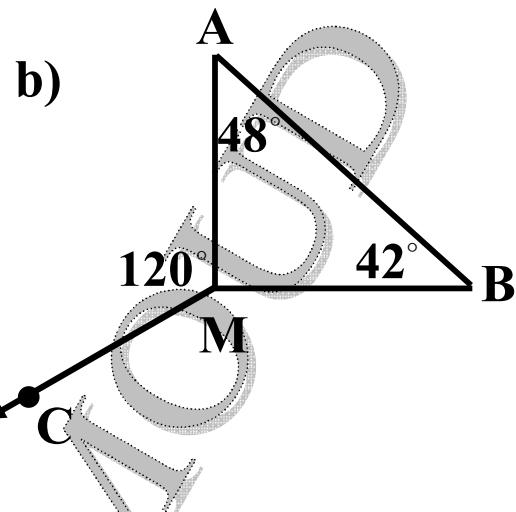
75) Find the measure of the required angle under each figure:



If  $M \in \overleftrightarrow{BD}$ ,  $\overrightarrow{MC}$  bisects

$\angle AMD$ , then

$m(\angle AMC) = \dots\dots$

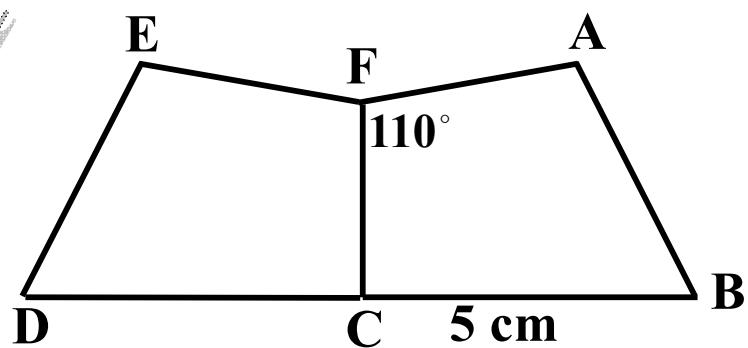


$m(\angle BMC) = \dots\dots$

76) In the opposite figure:

If  $C \in \overleftrightarrow{BD}$ ,

$m(\angle AFC) = 110^\circ$



$BC = 5 \text{ cm}$ . and the polygon  $ABCF \cong$  the polygon  $EDCF$

Complete the following :

a)  $\overline{CF}$  is ..... side.

b)  $BD = \dots \text{cm.}$

c)  $m(\angle FCD) = \dots \text{.....}$

d)  $m(\angle AFE) = \dots \text{.....}$

77) In the opposite figure:

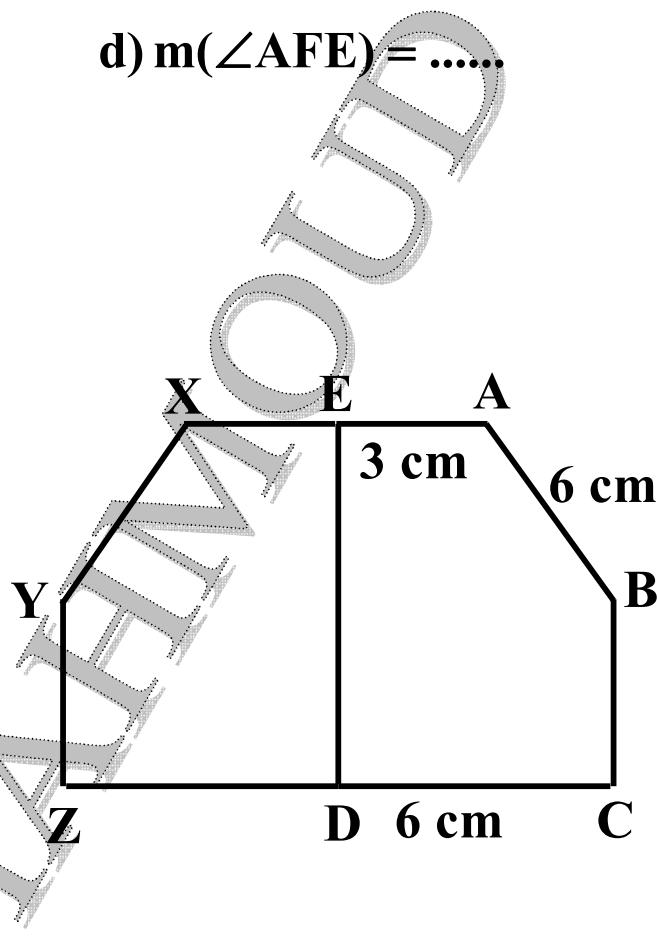
If  $D \in CZ$ , the figure

$ABCDE \cong$  the figure

$XYZDE$ ,  $AE = 3 \text{ cm}$

$BC = 4 \text{ cm}$  and

$AB = CD = 6 \text{ cm.}$



Complete the following:

a)  $XY = \dots \text{cm.}$

b)  $YZ = \dots \text{cm.}$

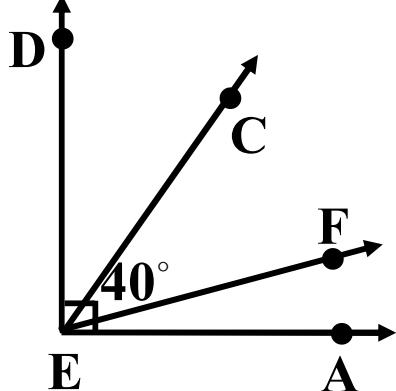
c)  $m(\angle EDC) = \dots \text{.....}$

d)  $CZ = \dots \text{.....cm.}$

e) The perimeter of the figure ABCZYX = .....cm.

78) Find the measure of the required angle under each figure:

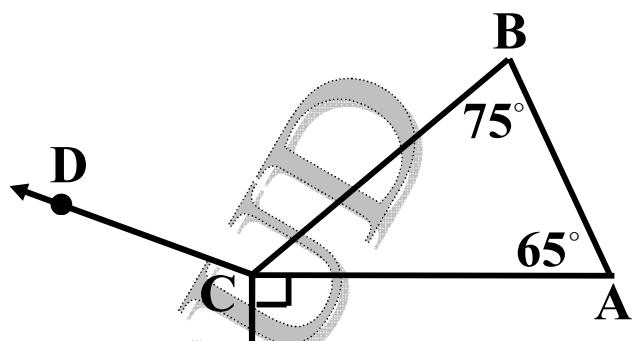
a)



If  $\overrightarrow{EA} \perp \overrightarrow{ED}$

then  $m(\angle AEF) = \dots\dots$

b)

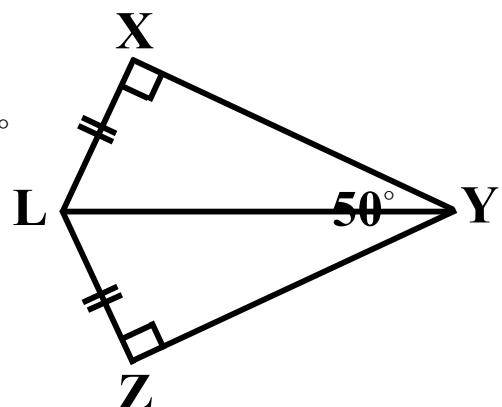


$m(\angle ECD) = \dots\dots$

79) In the opposite figure:

If  $LZ = LX$ ,  $m(\angle Z) = m(\angle X) = 90^\circ$

and  $m(\angle XYZ) = 50^\circ$



Complete the following:

a)  $\Delta XYL \cong \Delta \dots\dots$

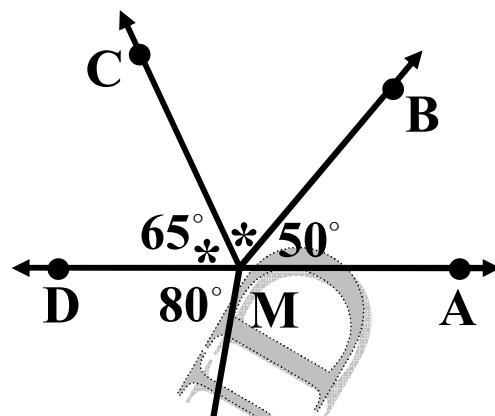
b)  $YZ = \dots\dots$

c)  $m(\angle XLY) = m(\angle \dots\dots) = \dots\dots$

80) Find the measure of the

required angle:

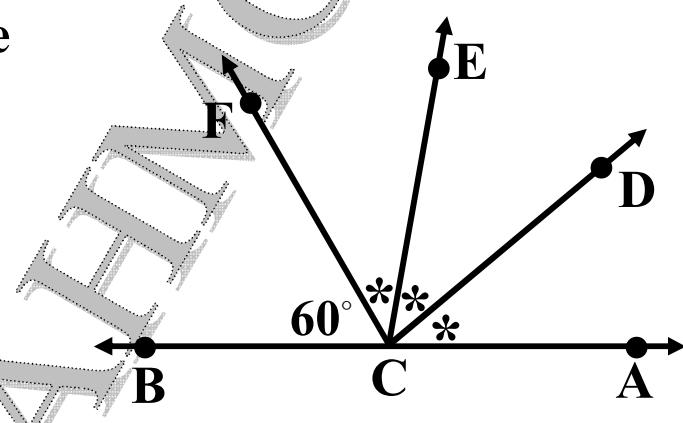
If  $\overrightarrow{MC}$  bisects  $\angle BMD$   
then  $m(\angle AME) = \dots\dots$



81) Find the measure of the

required angle:

If  $C \in \overleftrightarrow{AB}$  then  
 $m(\angle DCB) = \dots\dots$



82) In the opposite figure:

ABCD is a square of side length

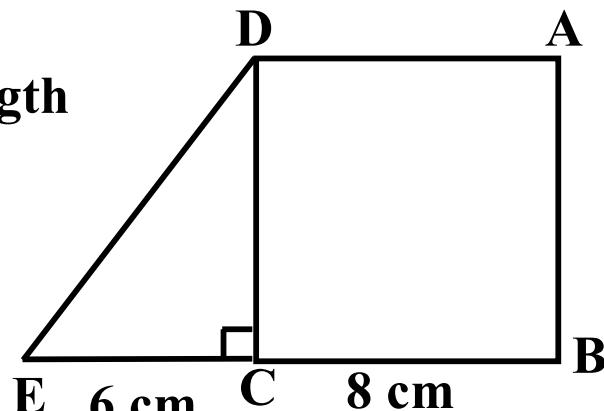
8 cm. and  $E \in \overrightarrow{BC}$  such that

$CE = 6 \text{ cm.}$

a) The area of  $\triangle DCE$

$= \dots\dots \text{cm}^2.$

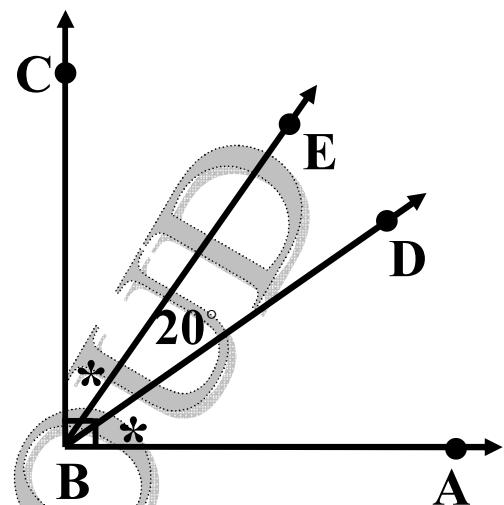
b) The area of the square set up  $\overline{DE} = \dots\dots \text{cm}^2.$



83) Find the measure of the required angle:

If  $\overrightarrow{BA} \perp \overrightarrow{BC}$

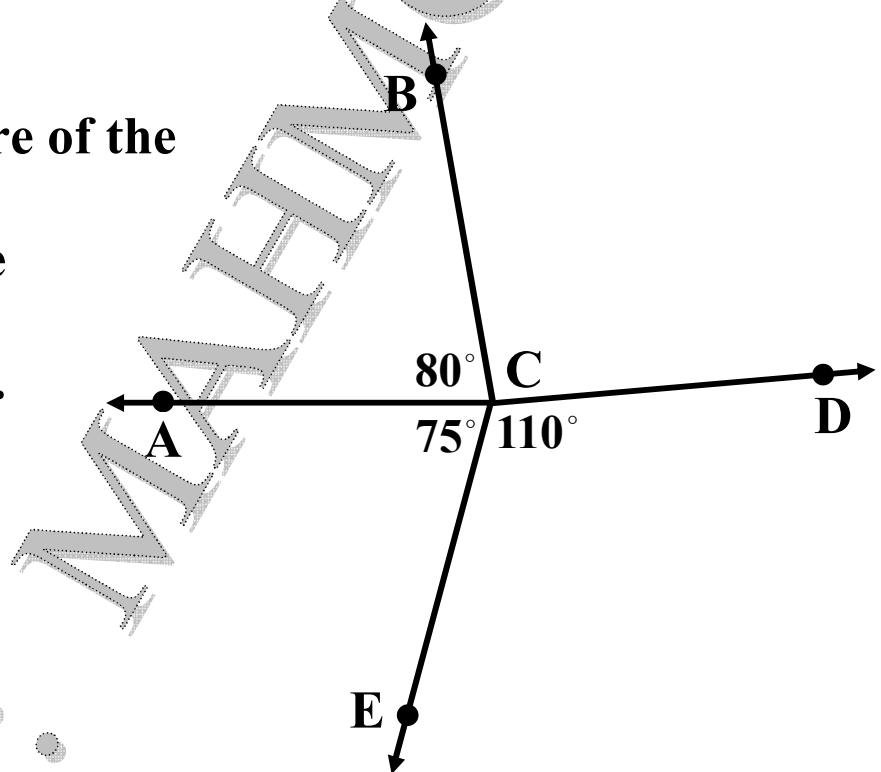
then  $m(\angle ABD) = \dots$



84) Find the measure of the

required angle

$m(\angle BCD) = \dots$



[2]

1) In the opposite figure:

**prep 1****GEOMETRY****FIRST TERM**

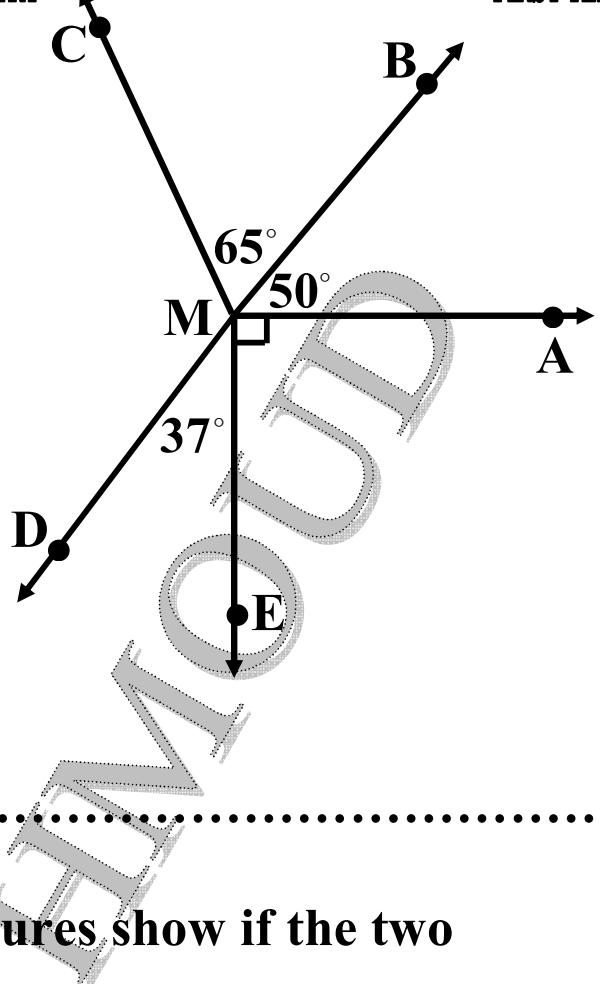
If  $m(\angle AMB) = 50^\circ$

$m(\angle BMC) = 65^\circ$

$m(\angle EMD) = 37^\circ$

and  $\overrightarrow{MA} \perp \overrightarrow{ME}$

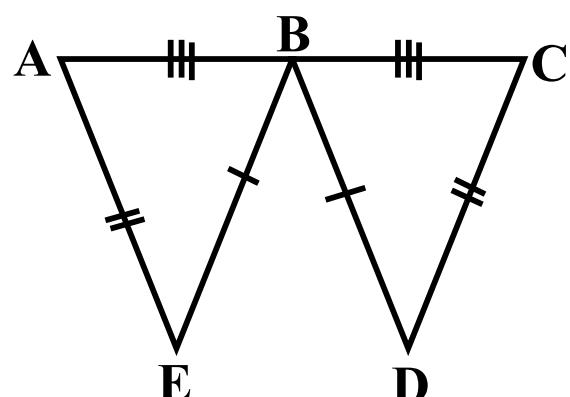
Find :  $m(\angle CMD)$

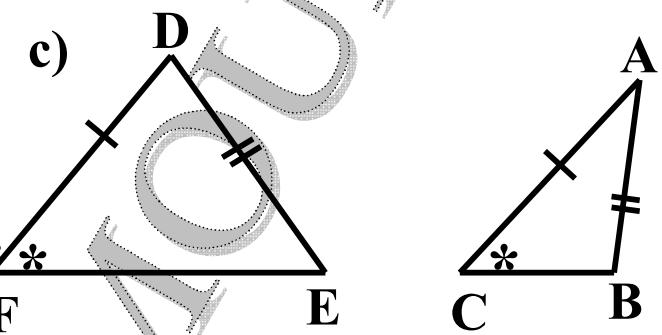
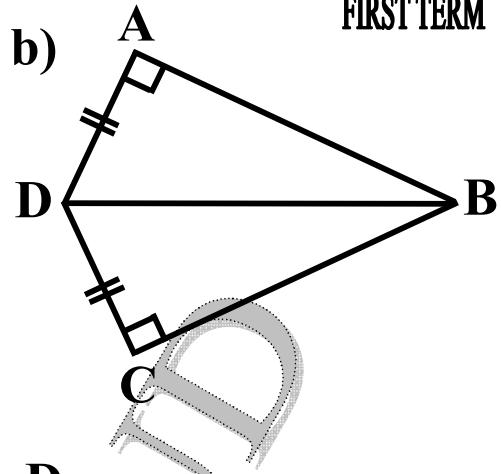


- 2) In each of the following figures show if the two triangles are congruent or not. If they are congruent name the case of congruency and state the results obtained. If they are not give reasons given that the similar signs denote to congruence of the labeled elements by them.

.....  
.....

a)



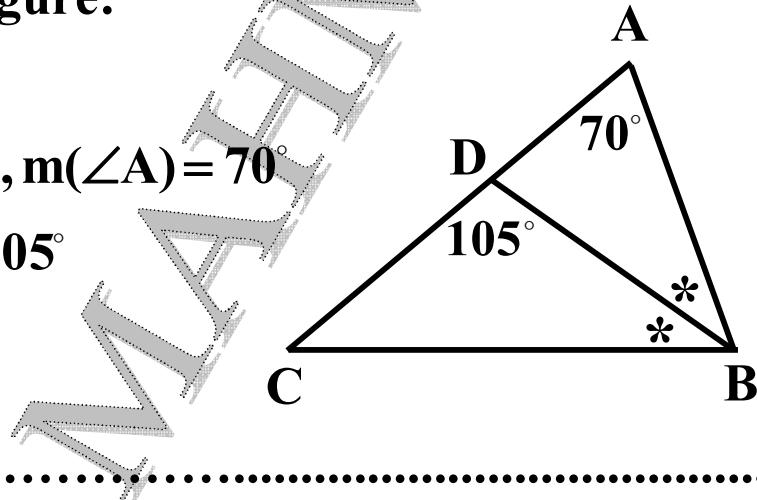


3) In the opposite figure:

$\overrightarrow{BD}$  bisects  $\angle ABC$ ,  $m(\angle A) = 70^\circ$

and  $m(\angle CBD) = 105^\circ$

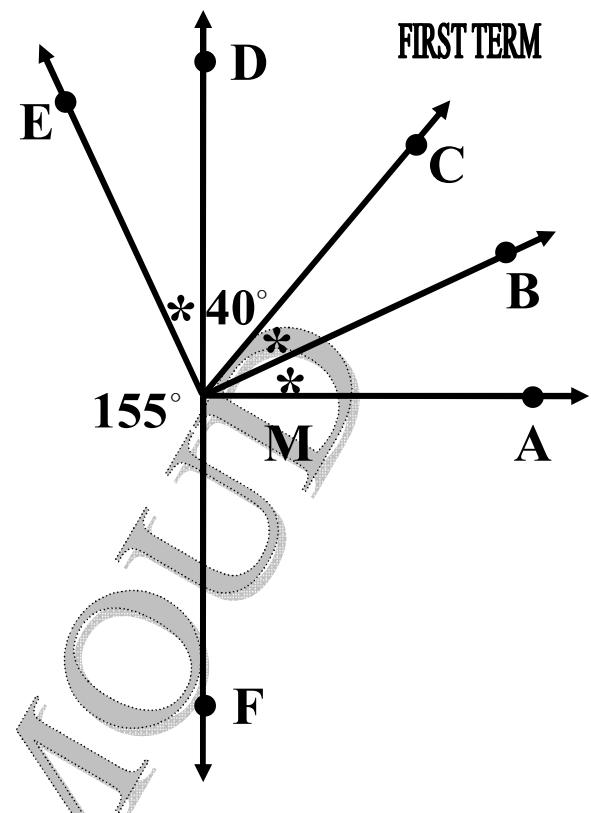
Find :  $m(\angle C)$



4) In the opposite figure:

If  $m(\angle AMB) = m(\angle BMC) = m(\angle DME)$  ,  $\overrightarrow{MA} \perp \overrightarrow{MF}$

$m(\angle EMF) = 155^\circ$  and  $m(\angle CMD) = 40^\circ$ . find:  $m(\angle AMB)$



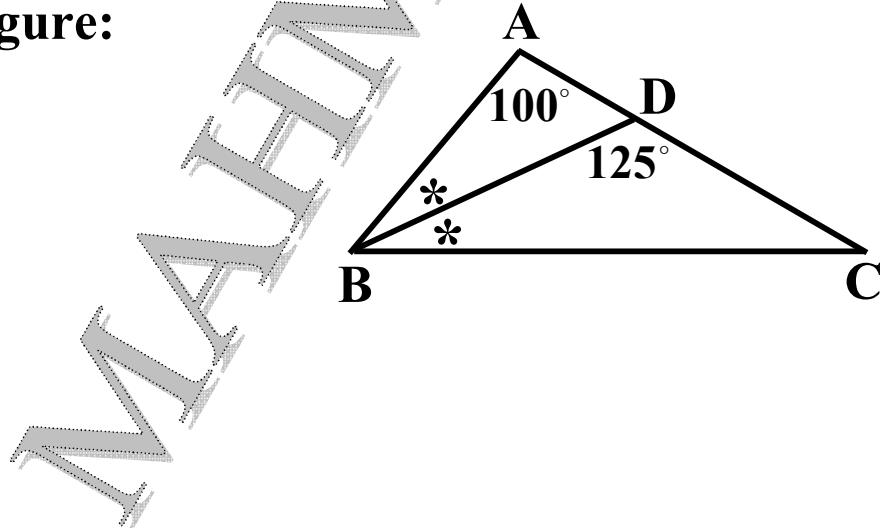
5) In the opposite figure:

$\overrightarrow{BD}$  bisects  $\angle CBA$

$m(\angle CDB) = 125^\circ$

$m(\angle A) = 100^\circ$

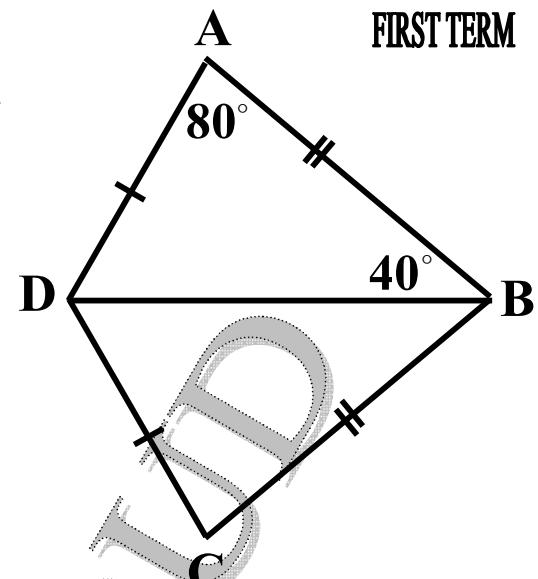
Find:  $m(\angle C)$



6) In the opposite figure:

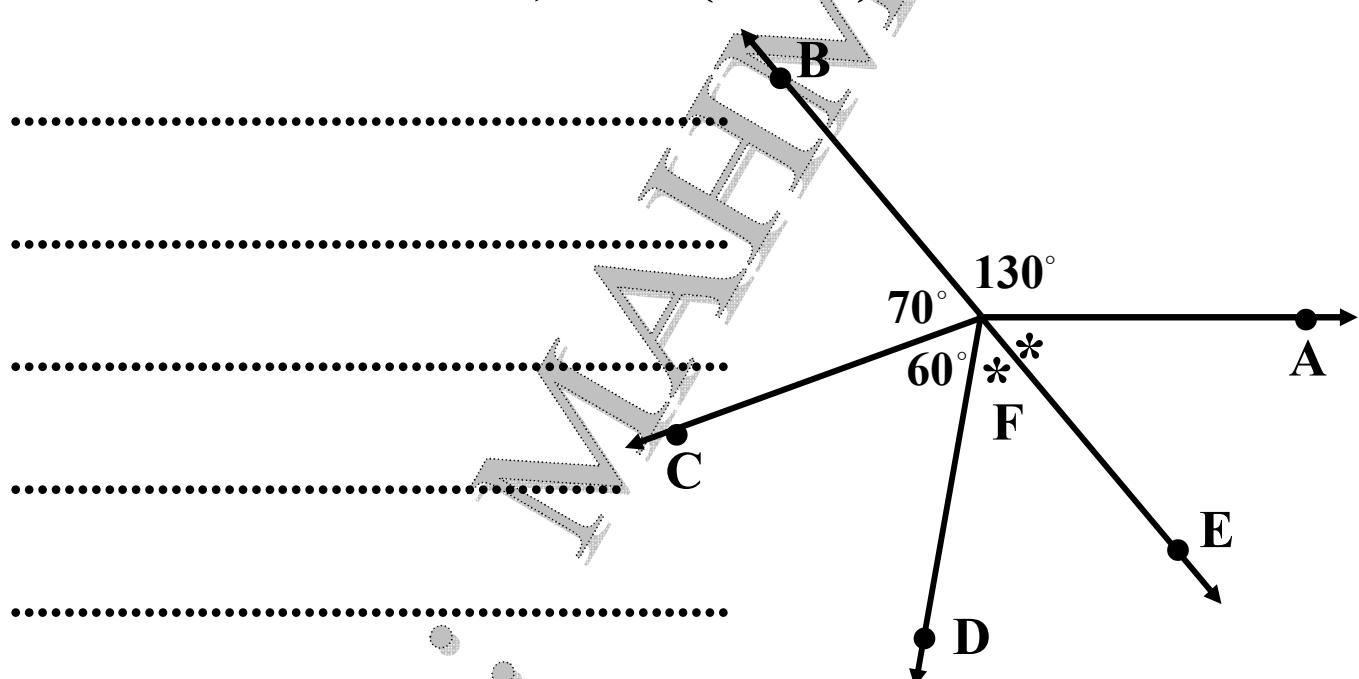
$AB = BC$ ,  $DA = DC$ ,  $m(\angle ABD) = 40^\circ$ ,

$m(\angle BAD) = 80^\circ$  find:  $m(\angle ADC)$

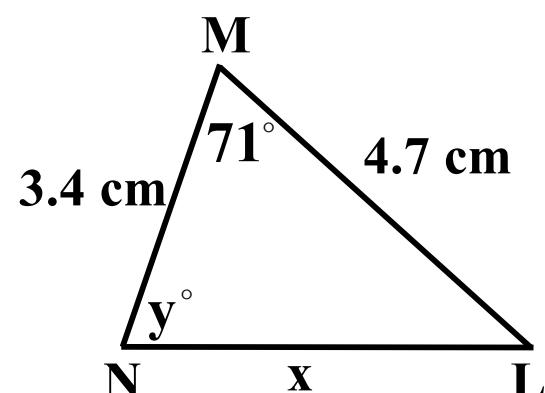
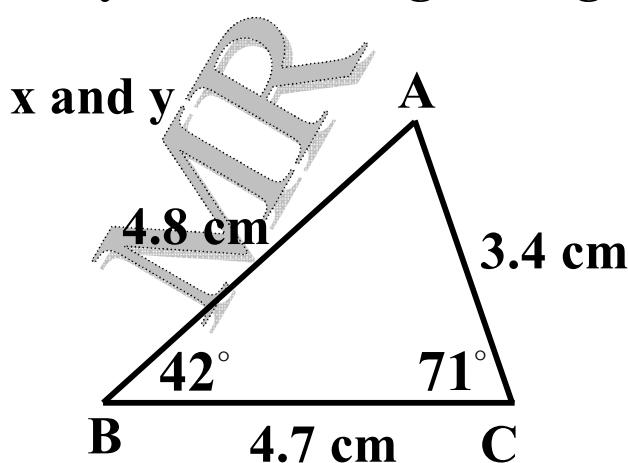


7) In the opposite figure:

If  $\overrightarrow{FE}$  bisects  $\angle AFD$ , find  $m(\angle AFE)$



8) Study the following two figures, then find the value of



.....  
.....  
.....  
**9) In the opposite figure:**

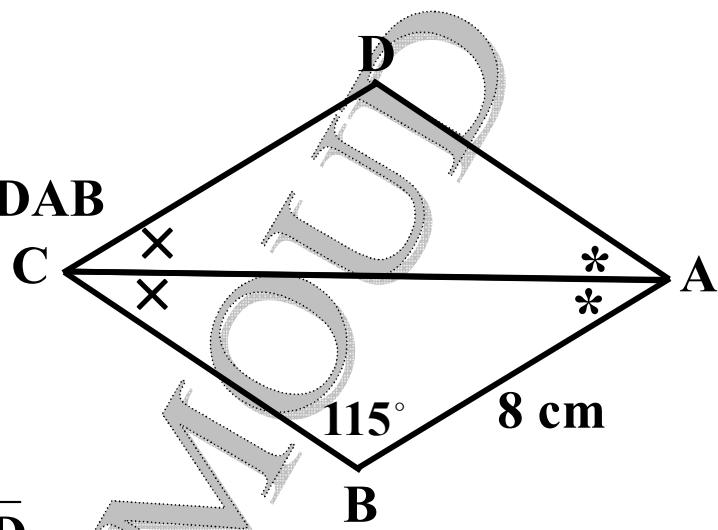
$\overleftrightarrow{AC}$  bisects  $\angle DCB$  and  $\angle DAB$

$m(\angle B) = 115^\circ$  and

$AB = 8 \text{ cm}$ .

Find : a)  $m(\angle D)$

b) The length of  $\overline{AD}$



.....  
.....  
**10) In each of the following figures show if the two**

**triangles are congruent or not. If they are**

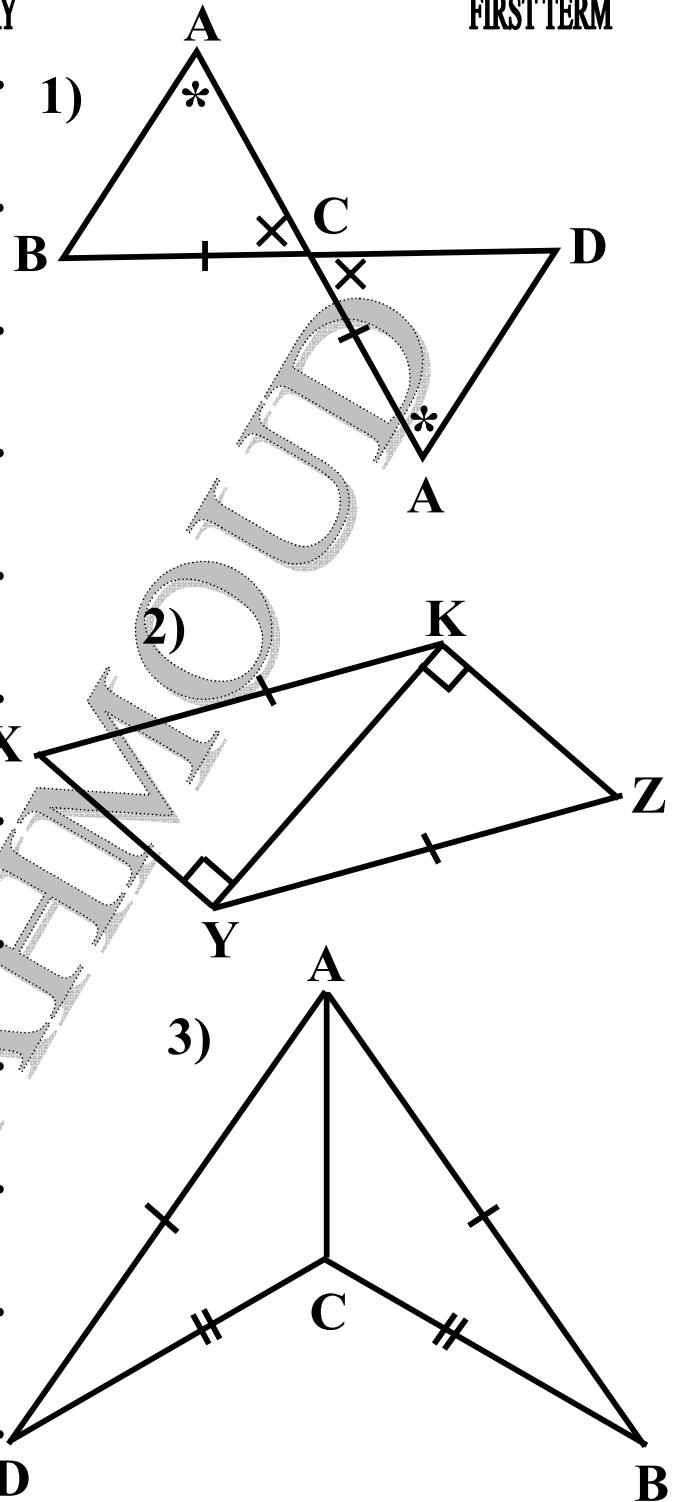
**congruent name the case of congruency and the**

**results of the congruency and if they are not**

**congruent give the reason. ( given that : the similar**

**signs denote to congruency of labeled elements by**

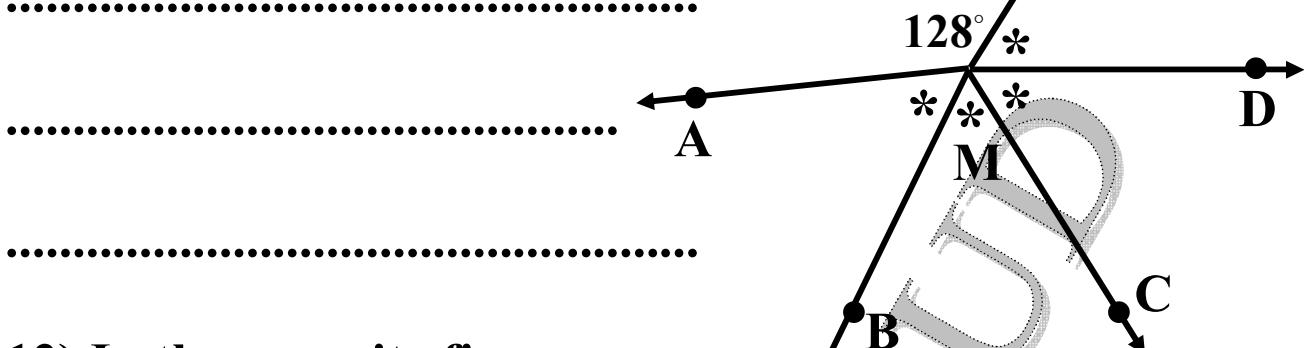
**these signs )**



11) In the opposite figure:

If  $m(\angle AMB) = m(\angle BMC) = m(\angle CMD) = m(\angle DME)$

and  $m(\angle EMA) = 128^\circ$ . find :  $m(\angle BMC)$



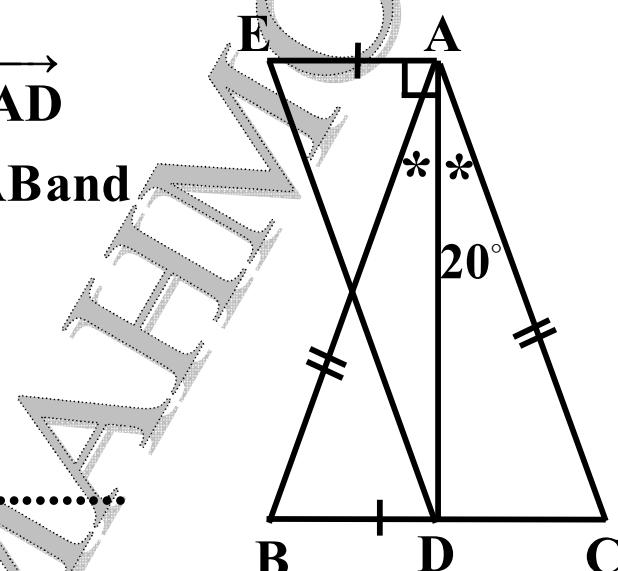
12) In the opposite figure:

$$AC = AB, AE = DB, \overrightarrow{AD}$$

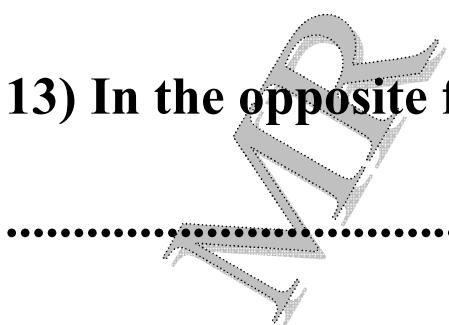
is the bisector of  $\angle CAB$  and

$$\overline{DA} \perp \overline{AE}$$

Find:  $m(\angle AED)$



13) In the opposite figure:



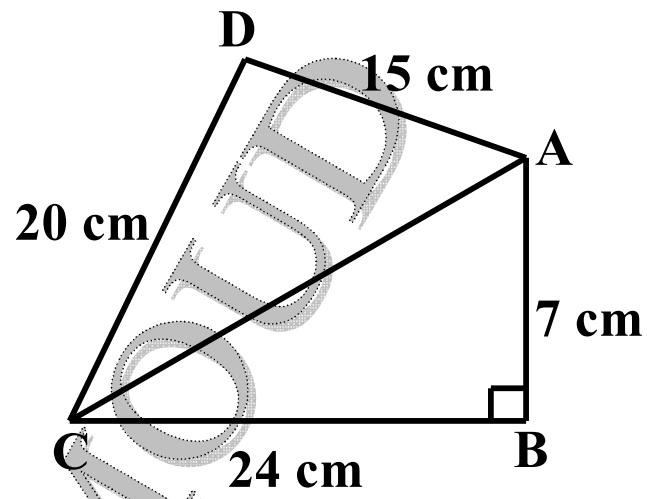
If  $m(\angle ABC) = 90^\circ$ ,  $AB = 7 \text{ cm}$ ,  $BC = 24 \text{ cm}$

$CD = 20 \text{ cm}$ . and  $DA = 15 \text{ cm}$ .

a) Is  $m(\angle ADC) = 90^\circ$  ?

Why?

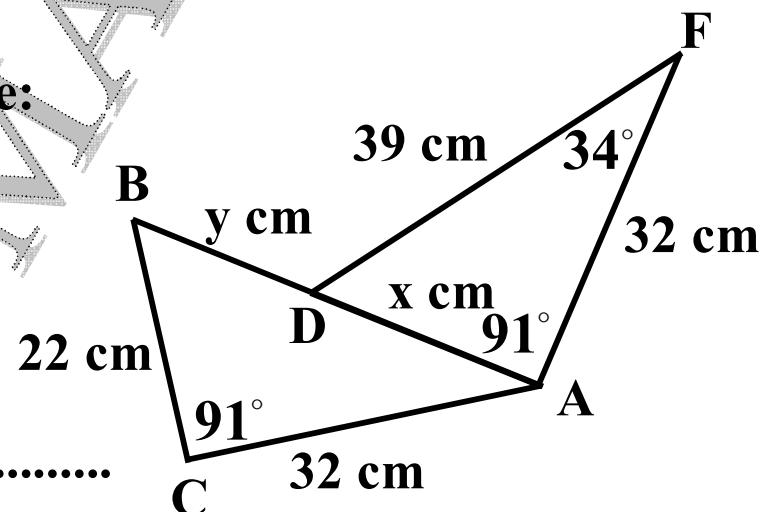
b) Find the area of the  
whole figure.



14) In the opposite figure:

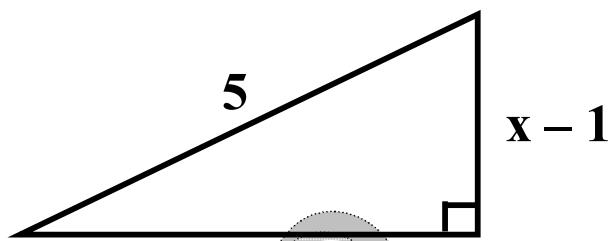
Find the value of

x and y



**[3] Choose:**

1) In the opposite figure:



Which of the following

represents a true statement?

a)  $x^2 + (x - 1)^2 = 25$

b)  $x + (x - 1) = 25$

c)  $x^2 - x = 12$

d)  $(x - 1)^2 - x^2 = 25$

2) Which of the following represents a correct

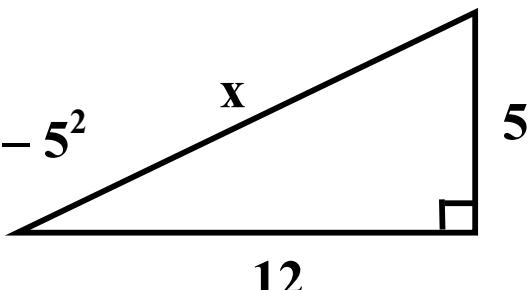
statement?

a)  $x = 5^2 + 12$

b)  $x^2 = 12^2 - 5^2$

c)  $x + 25 = 144$

d)  $x^2 = 169$

**[4]**

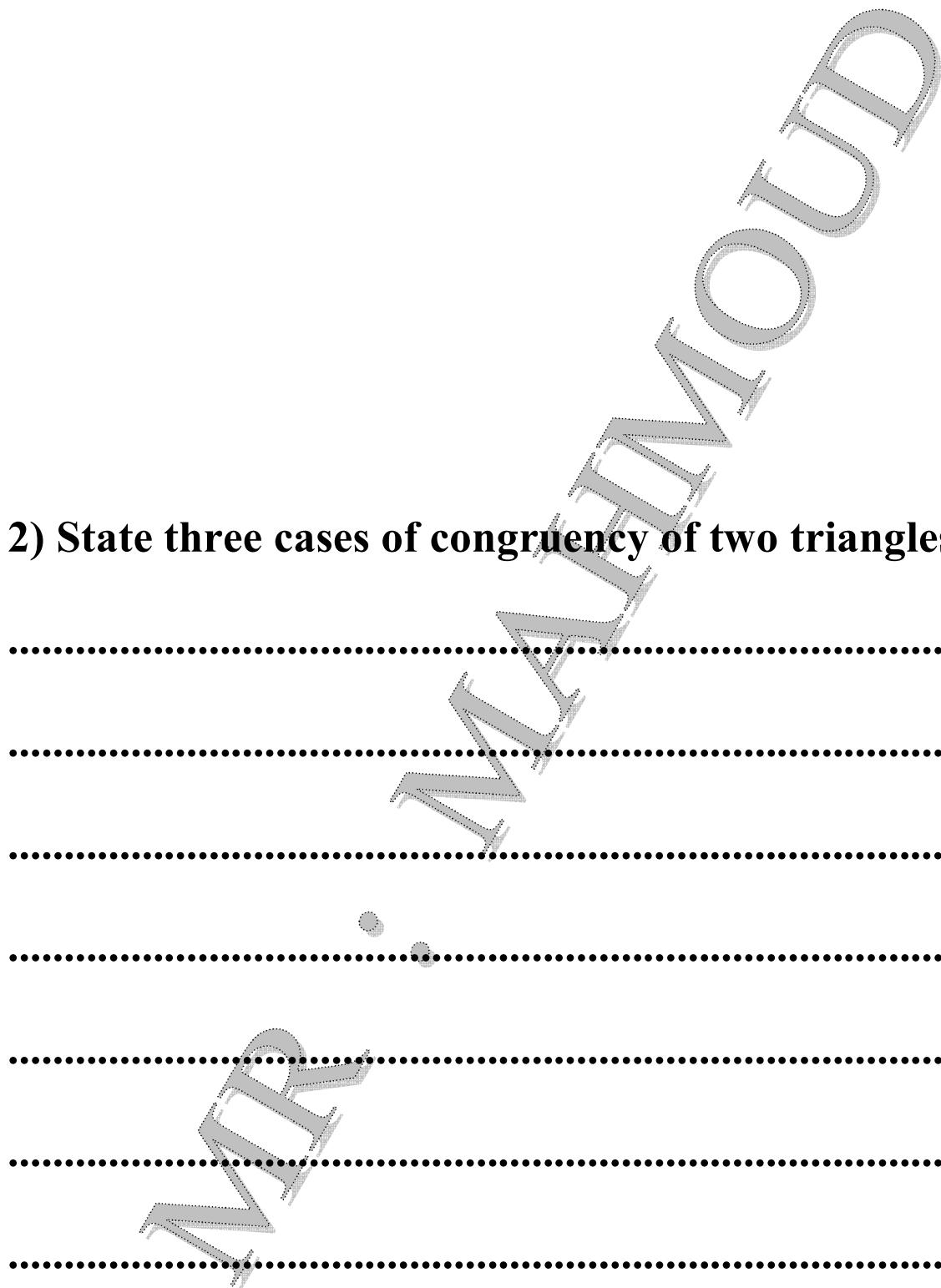
1) Using the ruler and compasses. Draw  $\triangle ABC$  in which

$AB = AC = 7 \text{ cm}$ . and  $BC = 6 \text{ cm}$ . then bisect each of

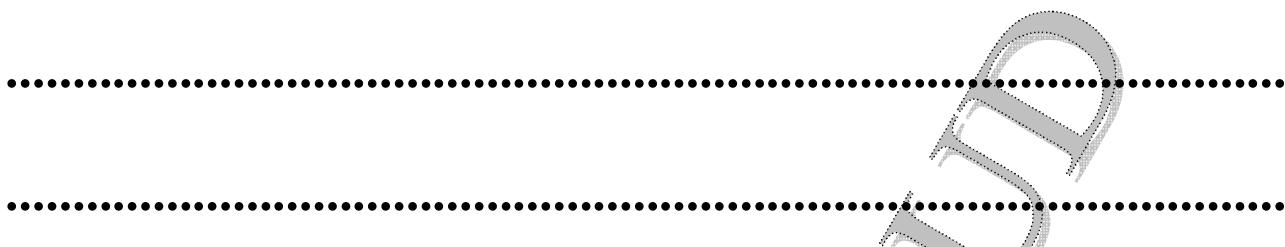
$\angle B$  and  $\angle C$  by two bisectors intersecting at M

Is  $MB = MC$ ? ( Don't remove arcs )

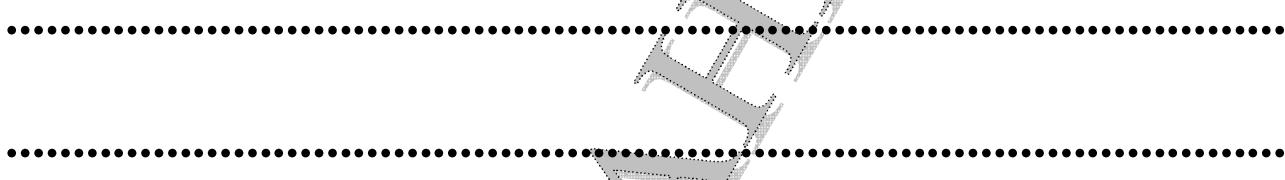
**2) State three cases of congruency of two triangles.**



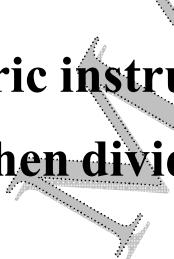
3) If  $\Delta ABC$  in which  $AB = 60 \text{ cm}$ ,  $AC = 61 \text{ cm}$ ,  $BC = 11 \text{ cm}$   
is  $\Delta ABC$  right - angled or not ? then if it is  
right - angled determine the right - angle.



4) In  $\Delta ABC$  :  $AB = 7 \text{ cm}$ ,  $AC = 25 \text{ cm}$  and  $BC = 24 \text{ cm}$ .  
is  $\Delta ABC$  is right - angled or not ?  
then determine the right angle if exist.



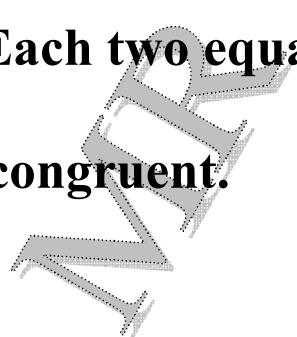
5) Using the geometric instruments draw an angle  
of measure  $130^\circ$  then divide it into 4 equal angles  
in measure



6) Put ( ✓ ) or ( ✗ ):-

a) Each two equal triangles in perimeter are

congruent.



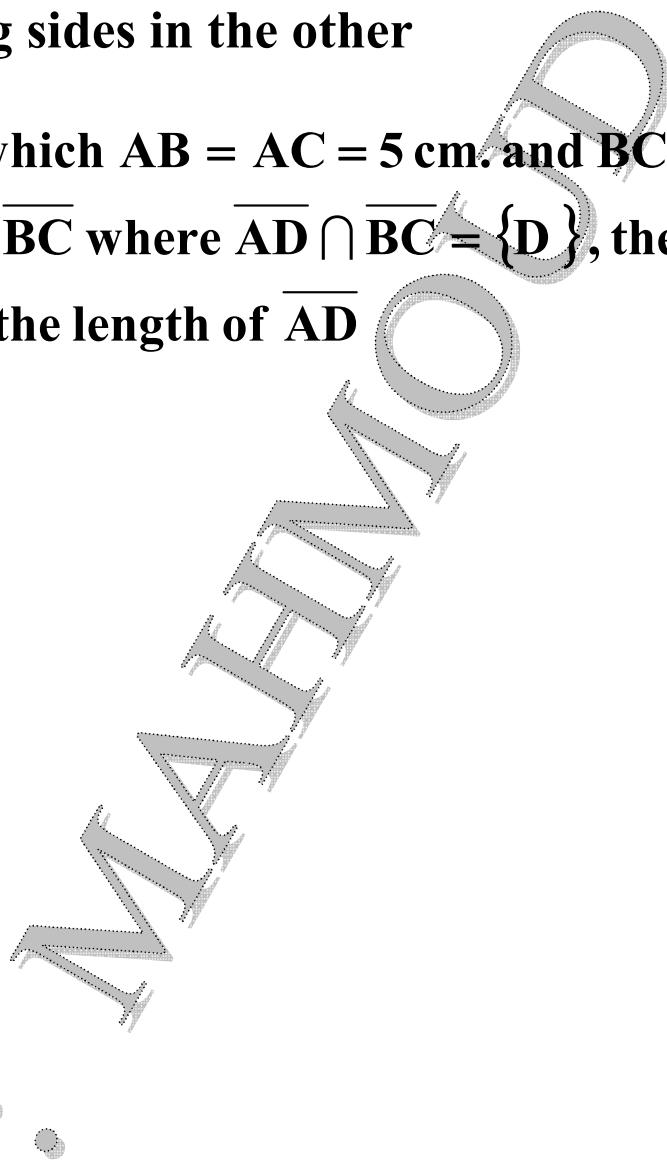
b) The two right-angled triangles are congruent if

two sides in one of them are equal to their

corresponding sides in the other

7) Draw  $\Delta ABC$  in which  $AB = AC = 5 \text{ cm}$ . and  $BC = 6 \text{ cm}$ .

then draw  $\overline{AD} \perp \overline{BC}$  where  $\overline{AD} \cap \overline{BC} = \{D\}$ , then  
find by measure the length of  $\overline{AD}$



8) A ladder of length 10 metre leans on a horizontal

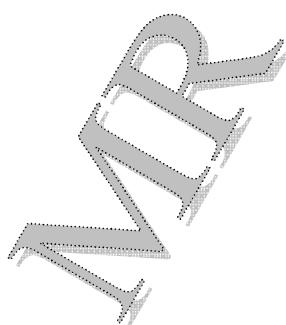
ground and a vertical wall if its upper end is over the

ground by 8 metre find the square of the distance

between its lower end and the wall. Show your work.

.....  
.....  
**9) LMN is a triangle in which  $LM = 20 \text{ cm}$ ,  $MN = 21 \text{ cm}$  and  $LN = 29 \text{ cm}$  show if  $\Delta LMN$  is a right-angled or not. determine the right angle if it exists.**

.....  
.....  
**10) Using the geometric tools to draw the equilateral triangle ABC whose side length = 4 cm. then draw  $\overline{AD} \perp \overline{BC}$  where  $\overline{AD} \cap \overline{BC} = \{D\}$**



11) Using the geometric tools to draw  $\triangle ABC$  in which  
 $AB = 3 \text{ cm}$ ,  $BC = 4 \text{ cm}$ ,  $AC = 5 \text{ cm}$ , then bisect  $\angle B$

by  $\overrightarrow{BD}$  which cuts  $\overline{AC}$  at D and find by measure the length of  $\overline{BD}$

12) Put (✓) or (✗):

a) Each two congruent triangles are equal in area.

b) The two triangles are congruent if the measures

of the angles of one of them are equal to its corresponding measures of the angles in the other two sides.

c) The area of the square set up a side of the right angle in the right-angled triangle equals the sum of the areas of the two squares set up the other two sides.

13) Draw  $\Delta ABC$  which is an equilateral and its side length = 5 cm. long then bisect  $\angle A$ ,  $\angle B$  and  $\angle C$  by three bisectors intersecting at M is  $MA = MB = MC$  ?

14) Draw  $\Delta ABC$  in which  $AB = 7 \text{ cm}$ ,  $m(\angle A) = 50^\circ$  and  $m(\angle B) = 70^\circ$ , then draw  $\overline{CD} \perp \overline{AB}$  and cut it at D, then find by measure the length of  $\overline{CD}$  then calculate the area of  $\Delta ABC$ .

**15) If ABC is a triangle in which  $AB = 3 \text{ cm}$ ,  $BC = 4 \text{ cm}$  and  $AC = 5 \text{ cm}$  show if the triangle ABC is right - angled or not ? then determine the right angle if it exist.**

**16) Using the geometric tools , draw an angle of measure  $65^\circ$  then bisect it.**

17) Draw  $\triangle ABC$  in which  $AB = AC = 6 \text{ cm}$ . and

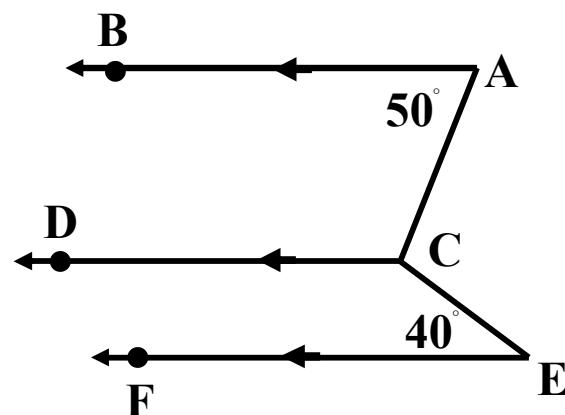
$m(\angle A) = 70^\circ$  using the compasses and the ruler to bisects  $\angle B$  and  $\angle C$  by two bisectors meeting at M

18) Mention two cases of congruency of two triangles.

[5] Complete:

1) In the opposite figure :

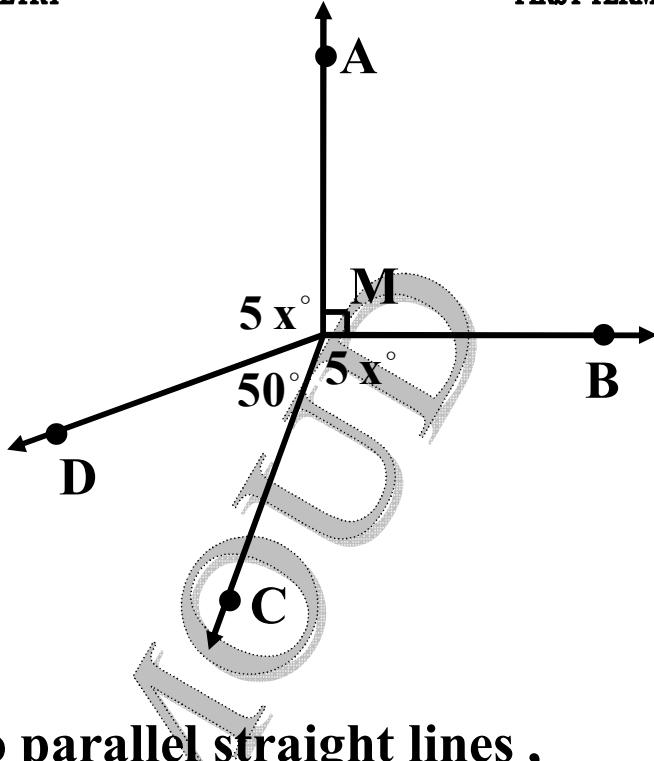
$$m(\angle ACE) = \dots\dots\dots$$



2) In the opposite figure :

the value of  $x = \dots\dots\dots$

.....  
.....  
.....



3) If a straight line cuts two parallel straight lines ,

then each two corresponding angles are

..... in measure.

4) The sum of the measures of the accumulative angles

at a point equals .....°.

.....  
.....

5) If two straight lines intersect, then each two vertical

opposite angles are ..... in measure.

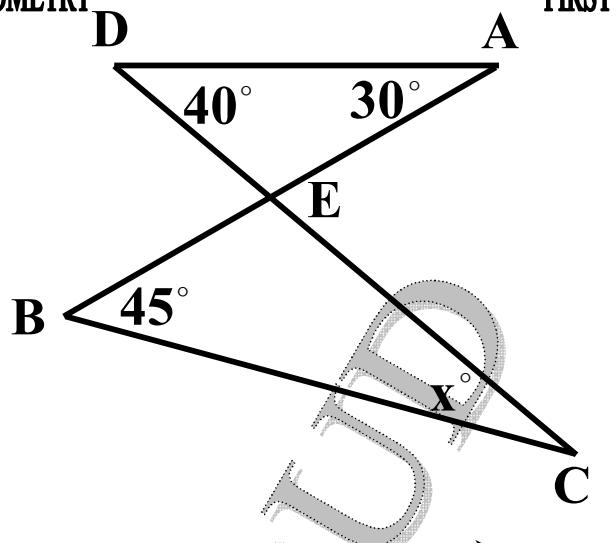
6) The perpendicular to one of two parallel

straight lines is ..... the other.

7) In the opposite figure :

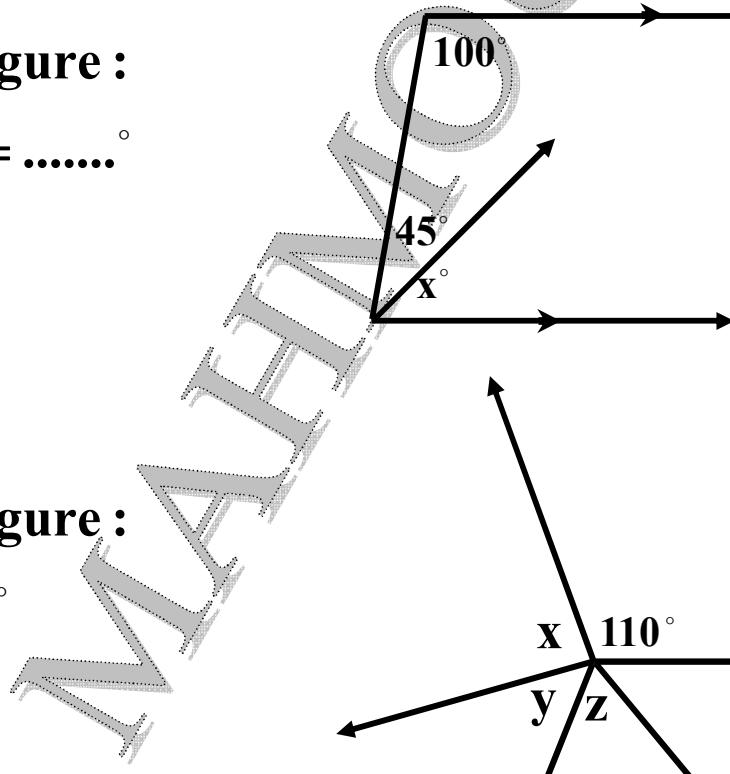
If  $\overline{AB} \cap \overline{CD} = \{E\}$

then  $x = \dots\dots\dots$



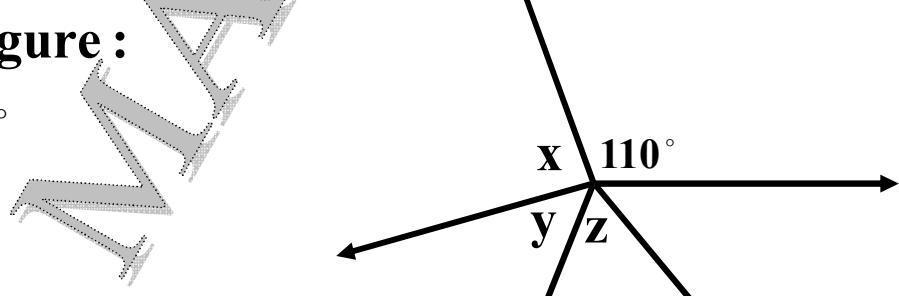
8) In the opposite figure :

The value of  $x = \dots\dots\dots^\circ$



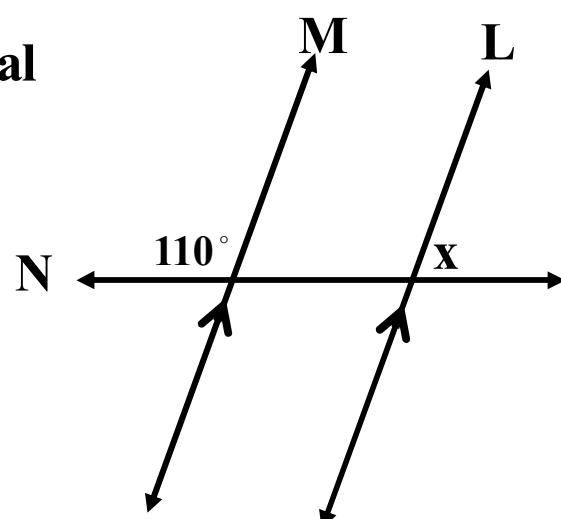
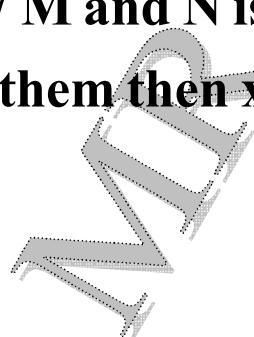
9) In the opposite figure :

$x + y + z = \dots\dots\dots^\circ$



10) In the opposite figure :

$L \parallel M$  and N is a transversal to them then  $x = \dots\dots\dots$



11) In the opposite figure :

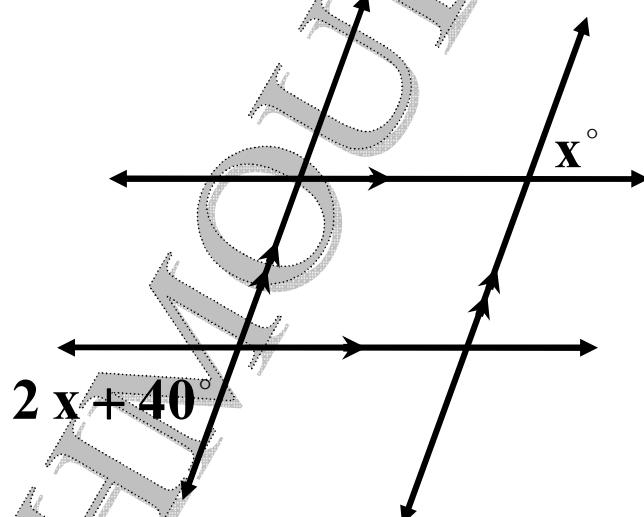
If  $L \parallel M$ , then the value of

$$x = \dots \dots \dots$$



12) In the opposite figure :

The value of  $x = \dots \dots \dots$

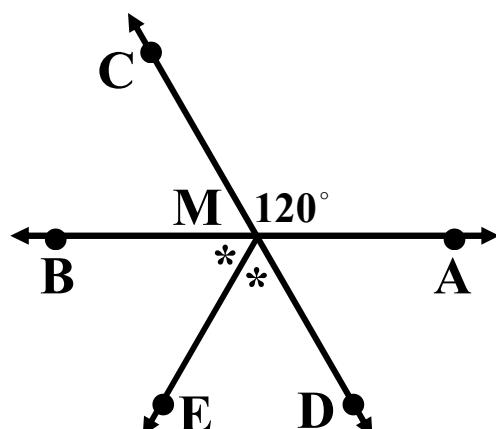


13) In the opposite figure :

$$\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{M\} \text{ and } \overrightarrow{ME}$$

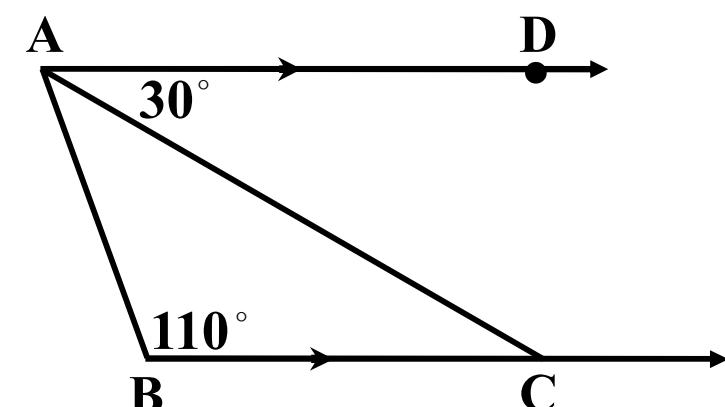
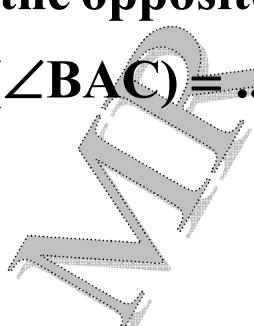
bisects  $\angle BMD$

$$\text{then } m(\angle AME) = \dots \dots \dots$$



14) In the opposite figure :

$$m(\angle BAC) = \dots \dots \dots$$

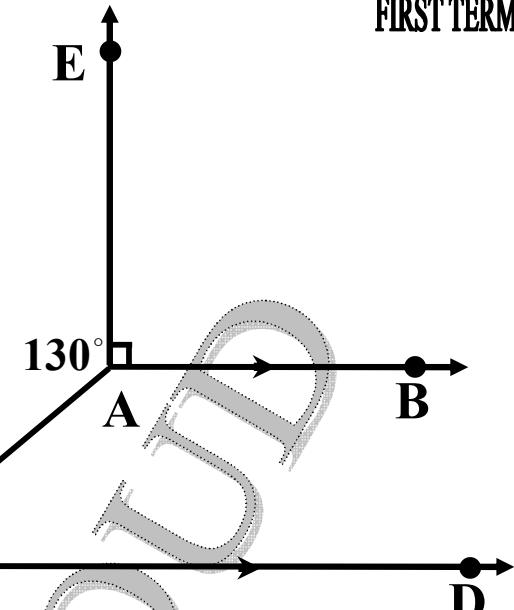


15) In the opposite figure :

If  $\overrightarrow{AB} \parallel \overrightarrow{CD}$ ,  $\overrightarrow{AE} \perp \overrightarrow{AB}$

and  $m(\angle EAC) = 130^\circ$

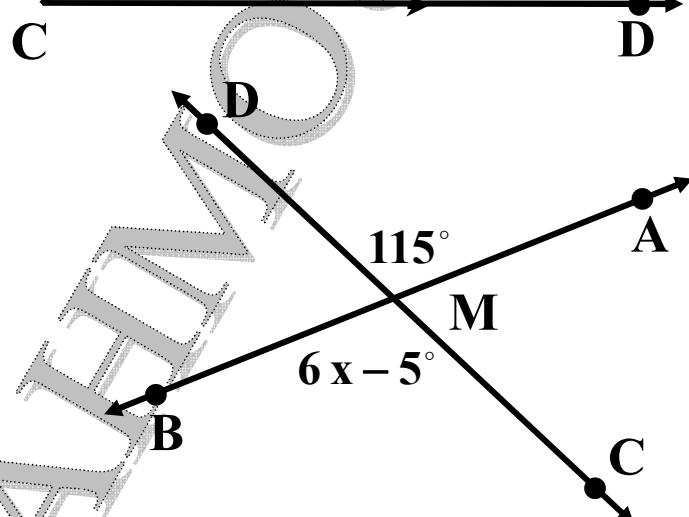
then  $m(\angle C) = \dots \dots \dots$



16) In the opposite figure :

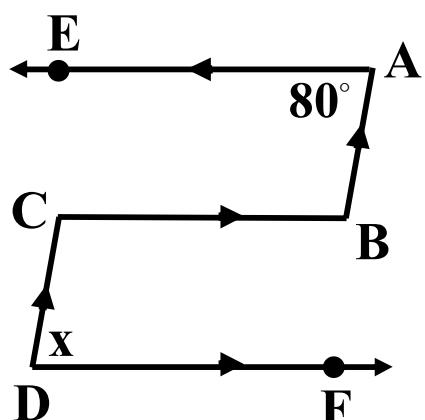
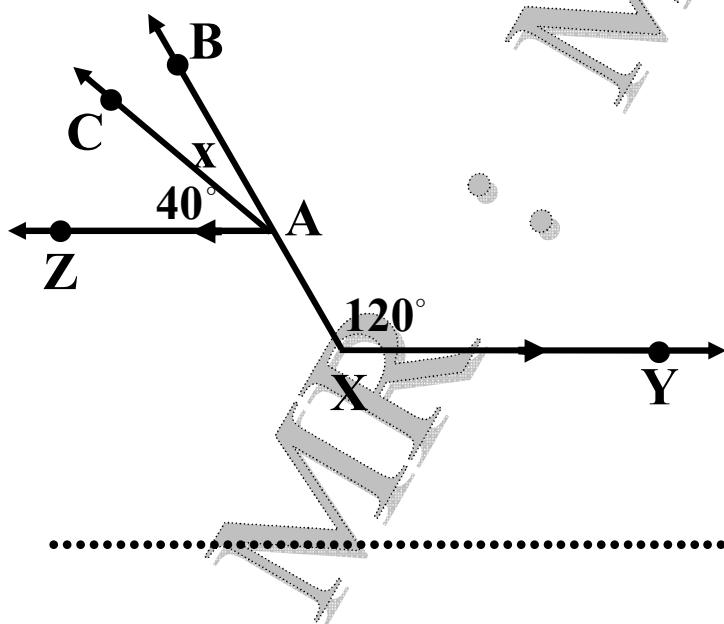
$\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{M\}$

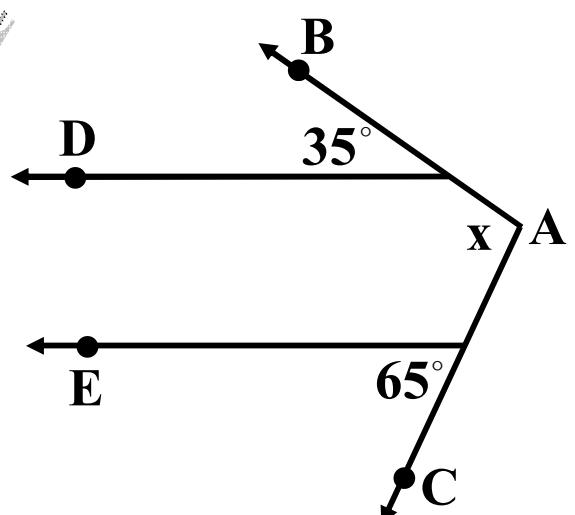
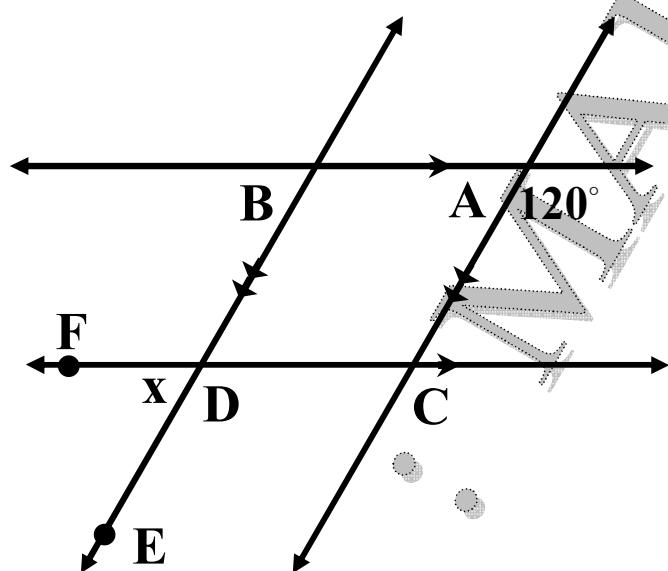
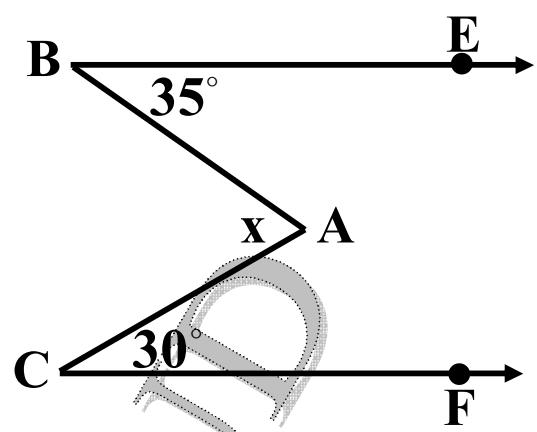
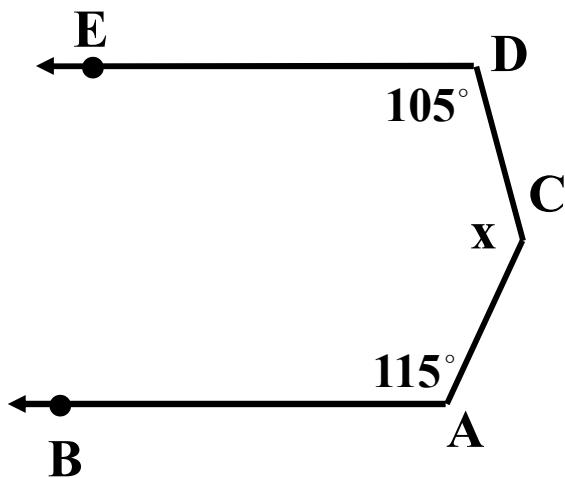
The value of  $x = \dots \dots \dots$



[6] In The opposite figure:

Find the value of  $x$





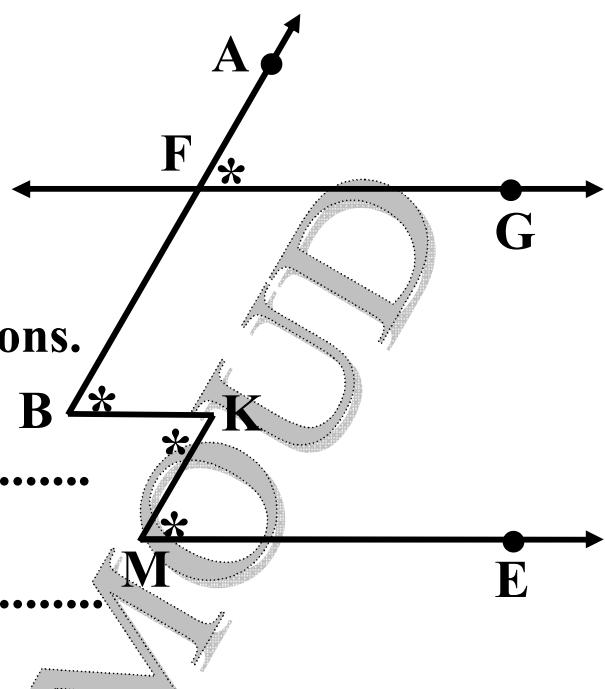
**[7]** In The opposite figure:

$$m(\angle AFG) = m(\angle B)$$

$$= m(\angle K) = m(\angle M)$$

write the four pairs of

parallel lines. give your reasons.



**[8]** In each of the following figures, show if the two

triangles are congruent or not if they are congruent  
name the case of congruence.

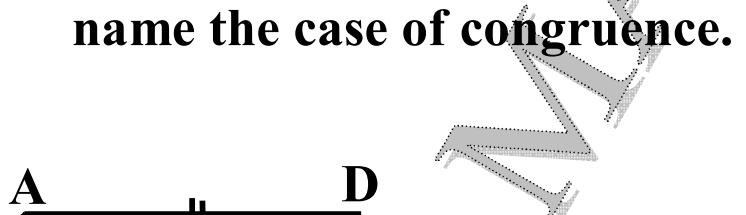


Figure (1)

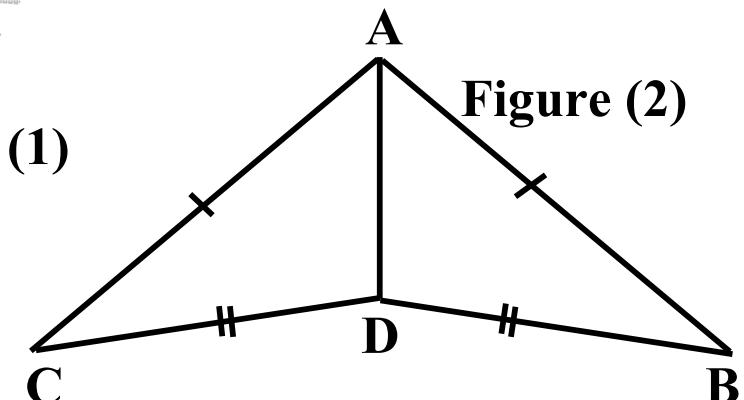
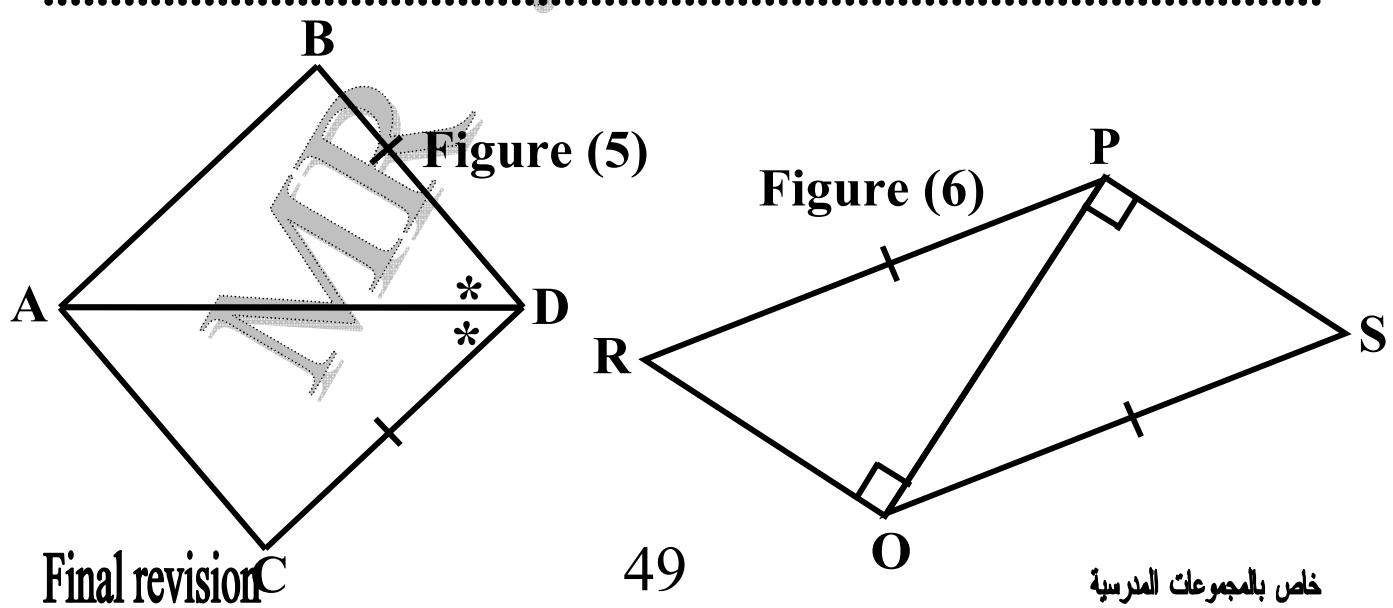
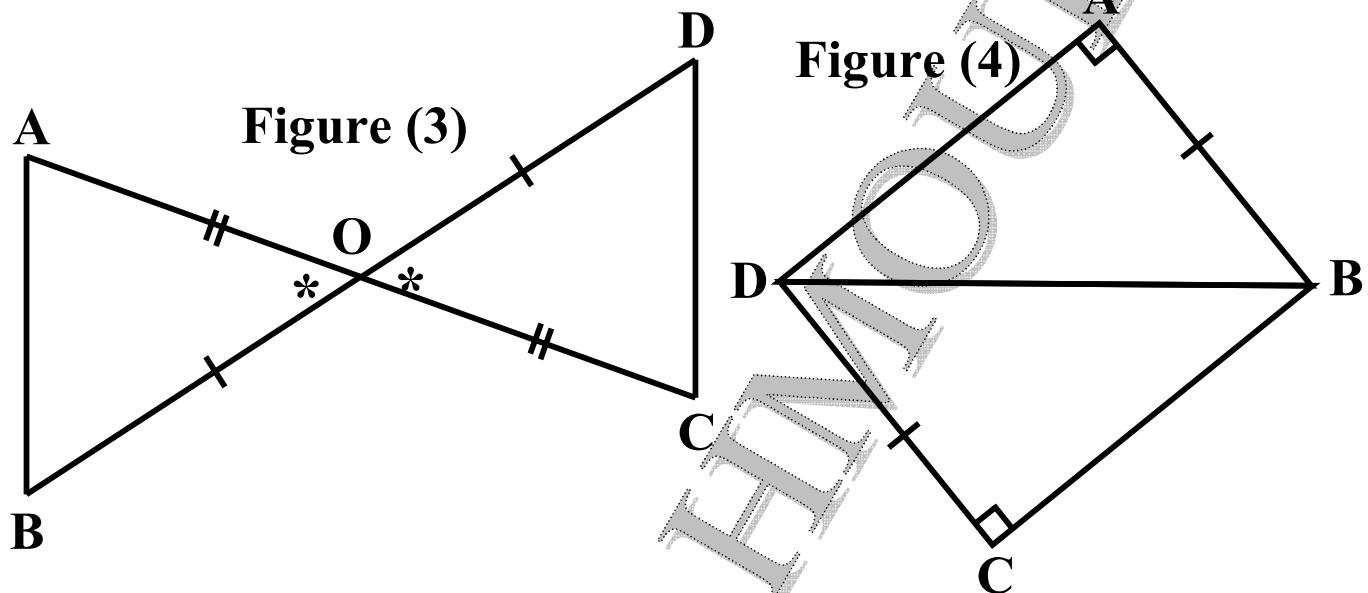
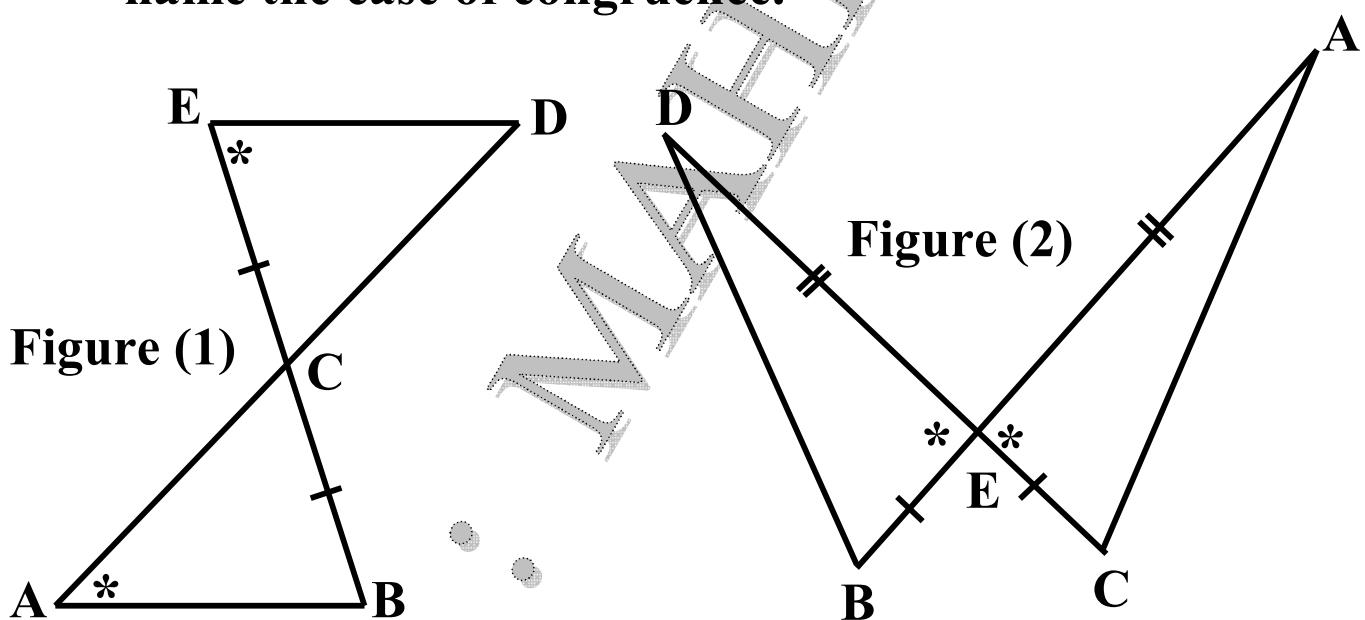


Figure (2)



[9] In each of the following figures , show if the two

triangles are congruent or not if they are congruent  
name the case of congruence.



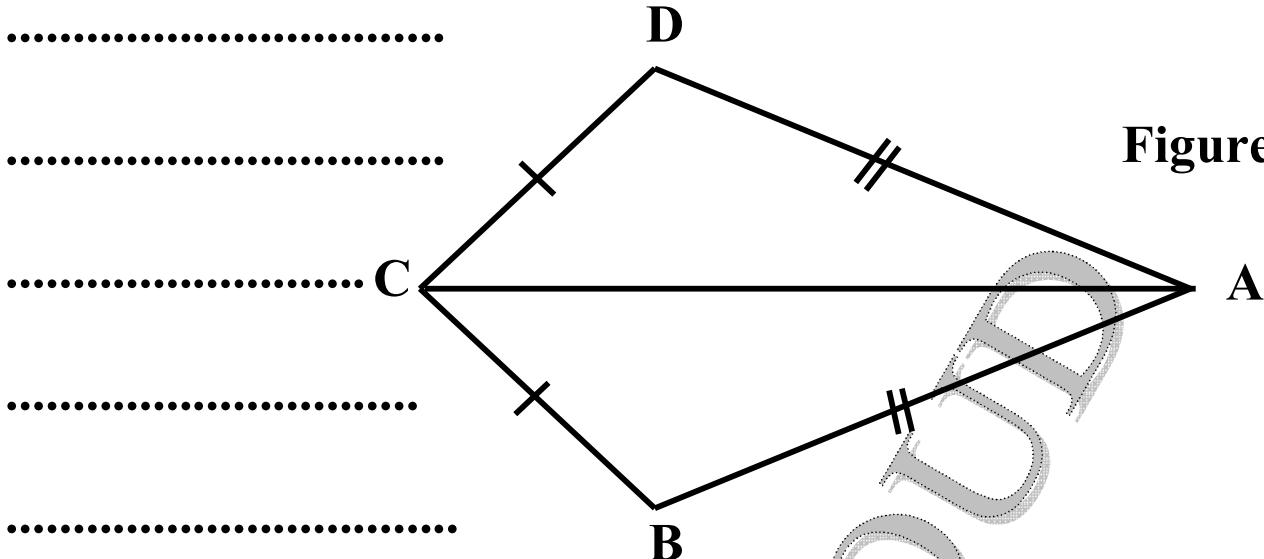


Figure (3)

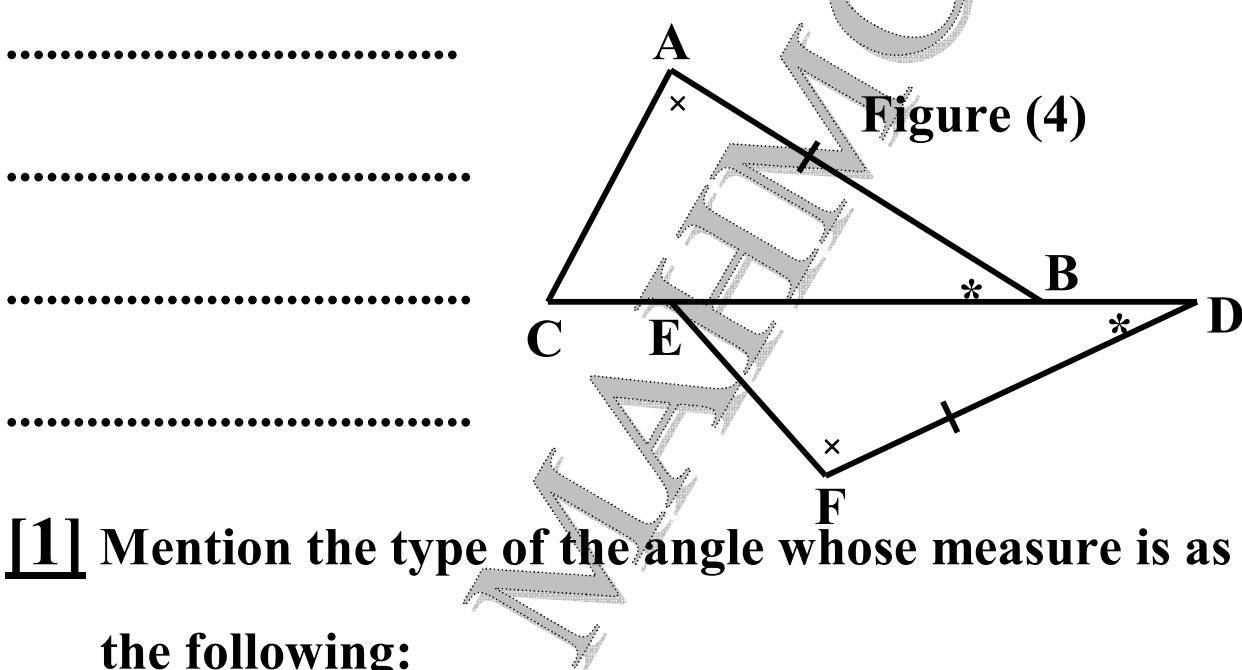


Figure (4)

**[1]** Mention the type of the angle whose measure is as the following:

1)  $57^\circ$  ..... 2)  $117^\circ$  ..... 3)  $90^\circ$  .....

4)  $200^\circ$  ..... 5)  $180^\circ$  ..... 6)  $90\frac{2}{5}^\circ$  .....

7)  $181\frac{3}{4}^\circ$  ..... 8)  $43\frac{1}{2}^\circ$  .....

**[2]** Write the measure of the angle which complements each of the angles whose measure are as follow:

1)  $20^\circ$  ..... 2)  $90^\circ$  ..... 3)  $152^\circ$  .....

4)  $0^\circ$  ..... 5)  $92\frac{1}{2}^\circ$  ..... 6)  $180^\circ$  .....

7)  $10^\circ$  ..... 8)  $141\frac{2}{5}^\circ$  .....

**[3]** Write the measure of the angle which supplements each of the angles whose measure are as follow:

1)  $30^\circ$  ..... 2)  $60^\circ$  ..... 3)  $48^\circ$  .....

4)  $0^\circ$  ..... 6)  $90^\circ$  .....

5)  $32\frac{1}{2}^\circ$  ..... 7)  $25\frac{3}{4}^\circ$  ..... 8)  $53\frac{1}{4}^\circ$  .....

**[4] Complete:**

1) The angle is .....

2) Measure of the right angle = .....

3) The acute angle is the angle whose measure is less

Than ..... and greater than .....

4) The sum of the complementary angles = .....

5) The sum of the supplementary angles = .....

6) Measure of the straight angle = .....

And the measure of zero angle = .....

7) The two adjacent angles formed from the

Intersection of a ray and a straight line are.....

### [5] Complete the following

1) The acute angle complements an.....angle and  
Supplements.....angle.

2) The zero angle complements a.....and  
Supplements a.....angle.

3) The right angle complements.....angle and  
Supplements a.....angle.

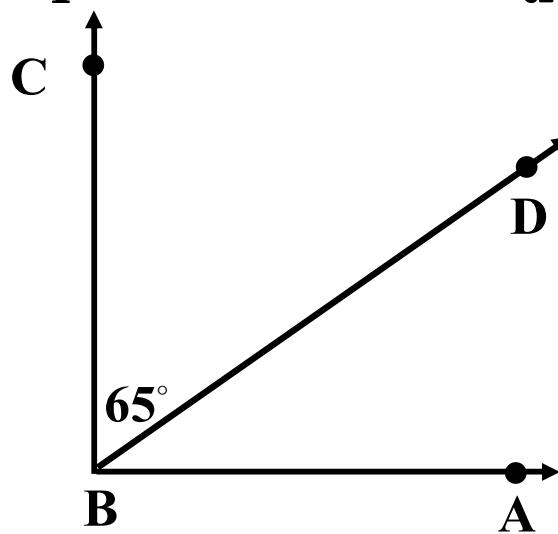
4) The obtuse angle supplements.....angle.

[6] In each of the following figures if  $\overrightarrow{BA} \perp \overrightarrow{BC}$  find the  
Measures of the required angle under each figure :

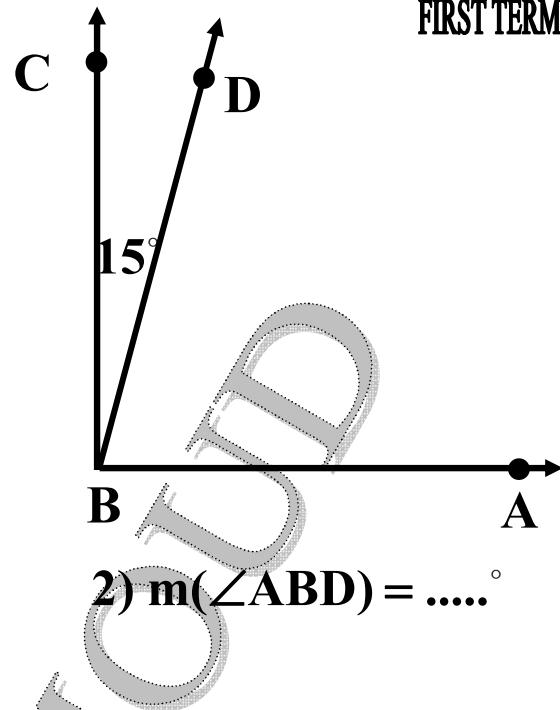
**prep 1**

**GEOMETRY**

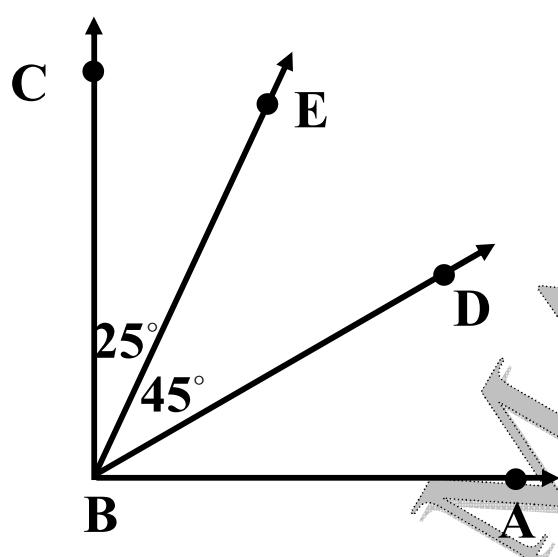
**FIRST TERM**



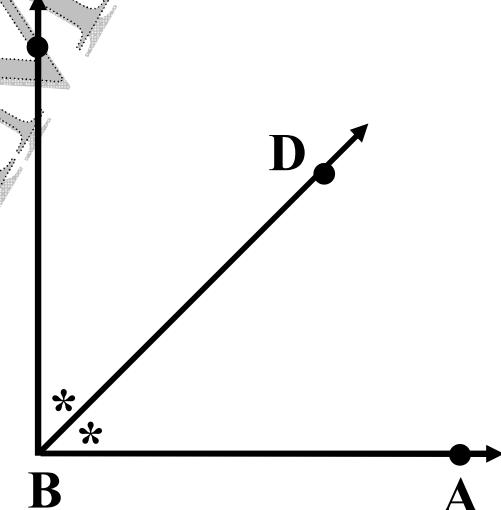
1)  $m(\angle ABD) = \dots \circ$



2)  $m(\angle ABD) = \dots \circ$



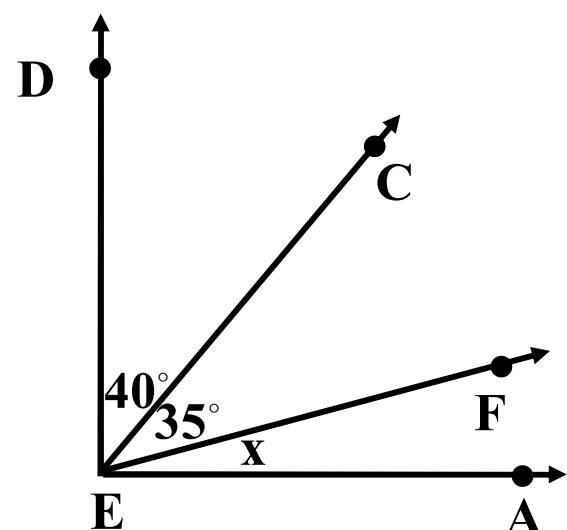
3)  $m(\angle ABD) = \dots \circ$



4)  $m(\angle ABD) = \dots \circ$

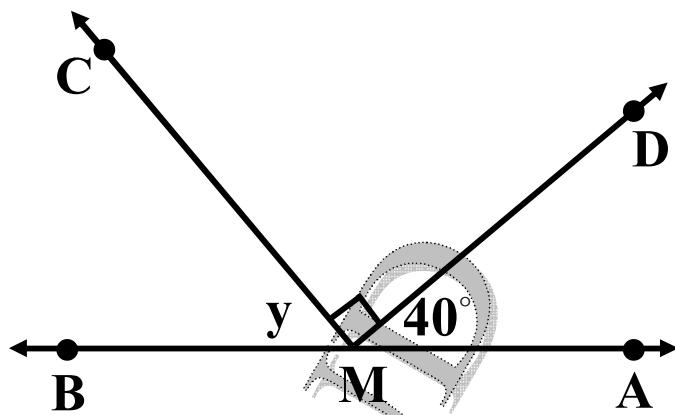
**[7] Complete:**

1) If  $EA \perp ED$   
Then  $x = \dots \circ$



2) If  $M \in \overleftrightarrow{AB}$

$$y = \dots \circ$$

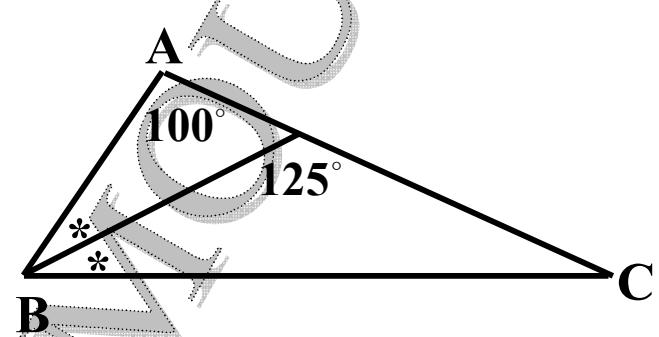


3) ABC is a triangle

$D \in \overline{AC}$  and  $\overrightarrow{BD}$  is

A bisector of  $\angle B$

Then  $m(\angle C) = \dots$

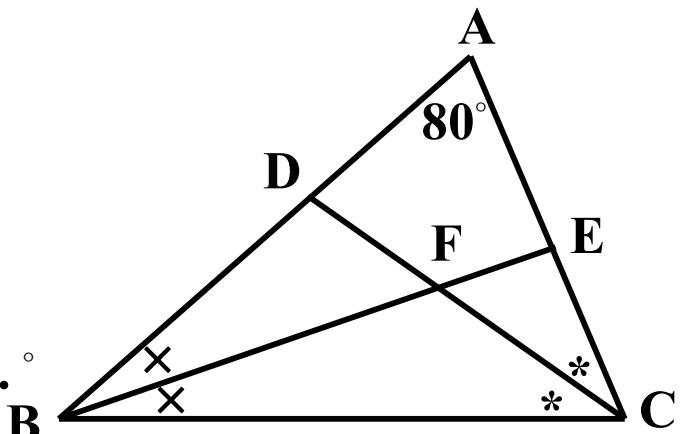


4)  $m(\angle A) = 80^\circ$ ,  $\overrightarrow{BE}$  is

The bisector of  $\angle B$

$\overrightarrow{CD}$  is the bisector of

$\angle C$  then  $m(\angle BFC) = \dots$   $\circ$



## Complete:

1) The sum of measures of the accumulative angles at a point = .....

2) The angle whose measure is  $72^\circ$  complements the angle whose measure is .....

3) If  $m(\angle A) = 150^\circ$ , then  $m(\text{reflex } \angle A) = .....$

4) The two adjacent complementary angles , their terminal sides are .....

5) If  $\angle A$  supplements  $\angle B$ ,  $\angle A \equiv \angle B$  , then  $m(\angle B) = .....$

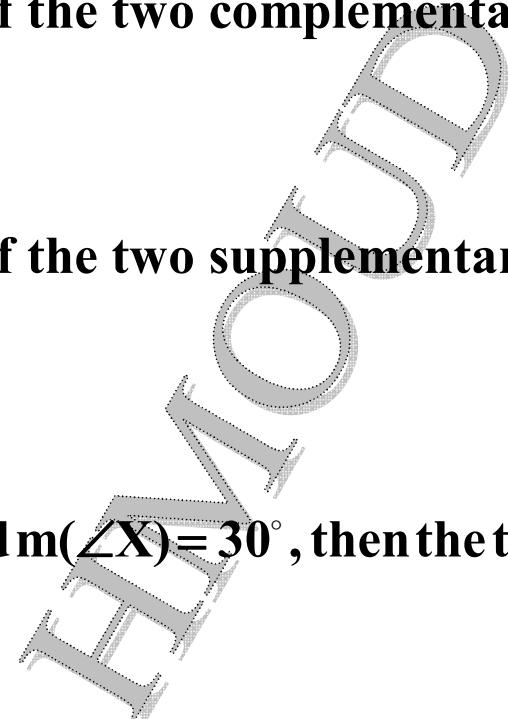
6) The measure of the straight angle = .....

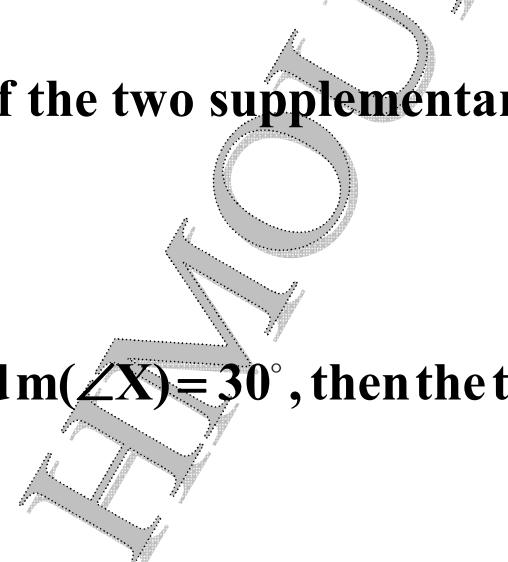
7) If one of the two supplement angles is acute then the other is ..... angle.

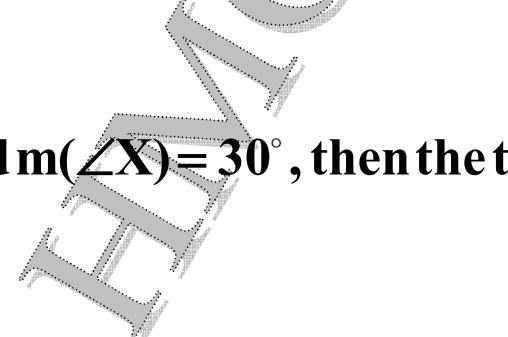
8) If  $m(\angle A) = 170^\circ$  , then  $m(\text{reflex } \angle A) = .....$

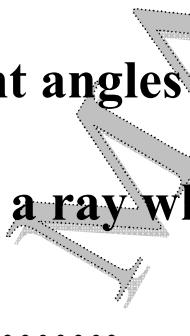
9) ..... < the measure of the obtuse angle < .....

10) If  $\angle A$  supplements  $\angle B$ , and  $m(\angle A) = 2 m(\angle B)$ ,  
then  $m(\angle B) = \dots\dots$

11) The sum of measures of the two complementary  
angles = .....  


12) The sum of measures of the two supplementary  
angles equals .....  


13) If  $m(\angle X) = \frac{1}{2}m(\angle Y)$  and  $m(\angle X) = 30^\circ$ , then the two  
angles X and Y are.....  


14) The two adjacent angles formed by intersecting a  
straight line and a ray whose start point lies on the  
straight line are .....  


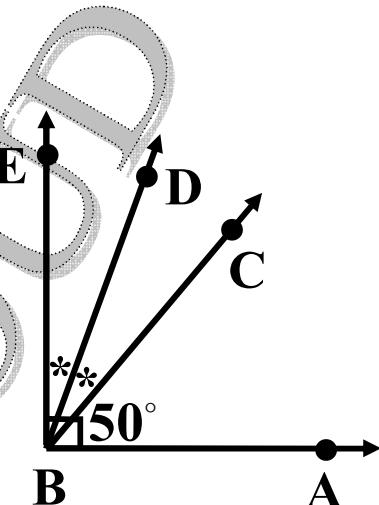
**Complete:**

- 1) If  $m(\angle X) = \frac{1}{2} m(\angle Y)$  and  $m(\angle X) = 60^\circ$ , then the two angles X and Y are .....

- 2) In the opposite figure:

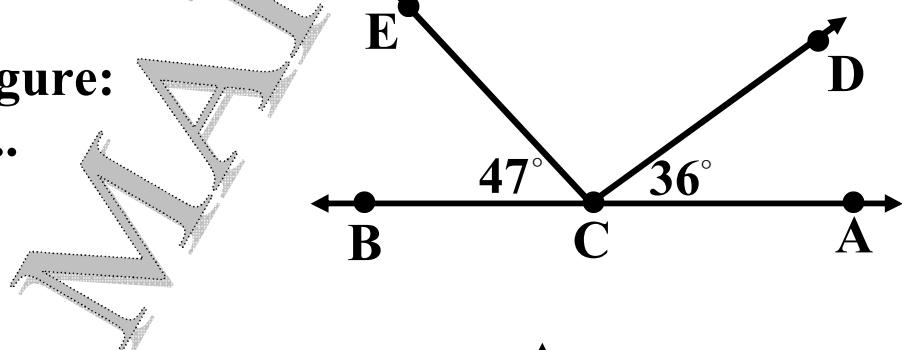
If  $m(\angle ABC) = 50^\circ$ ,  $\overrightarrow{BD}$  bisects  $\angle CBE$

$\overrightarrow{BD} \perp \overrightarrow{BE}$ , then  $m(\angle CBD) = \dots$



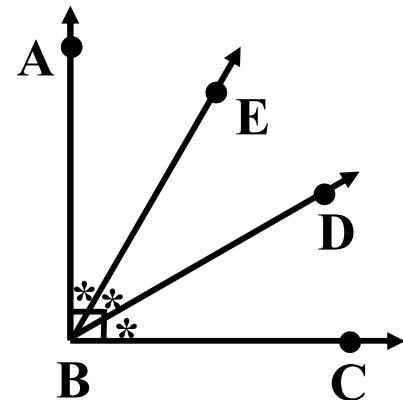
- 3) In the opposite figure:

$m(\angle DCE) = \dots$



- 4) In the opposite figure:

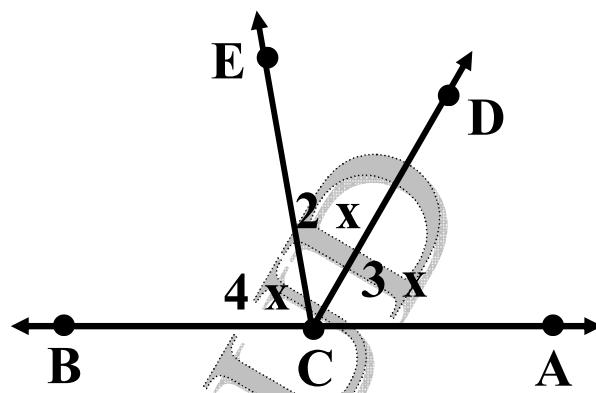
If  $\overrightarrow{BA} \perp \overrightarrow{BC}$ , then  $m(\angle CBE) = \dots$



Test (7)Complete:

1) In the opposite figure:

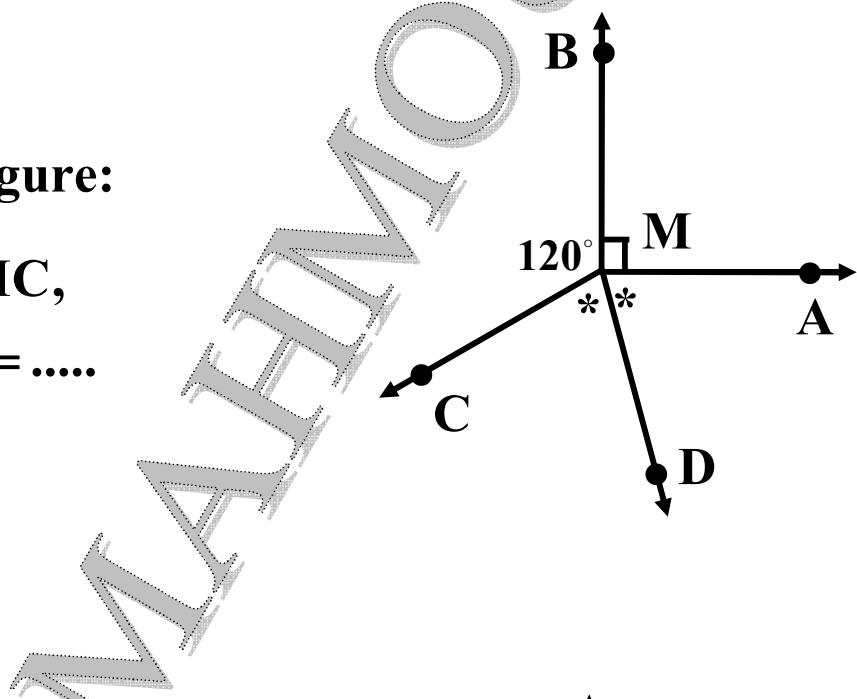
If  $A \in BC$ , then  $x = \dots$



2) In the opposite figure:

$\overrightarrow{MD}$  bisects  $\angle AMC$ ,

then  $m(\angle AMD) = \dots$

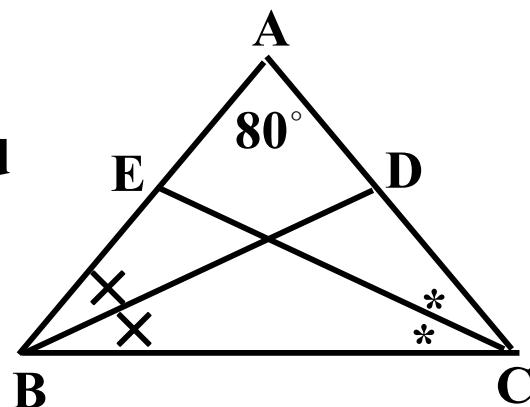
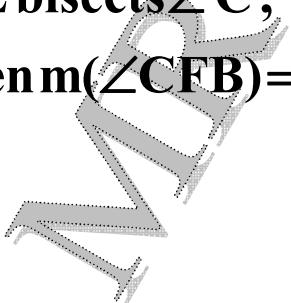


3) In the opposite figure:

$m(\angle A) = 80^\circ$ ,  $\overrightarrow{BD}$  bisects  $\angle B$  and

$\overrightarrow{CE}$  bisects  $\angle C$ ,

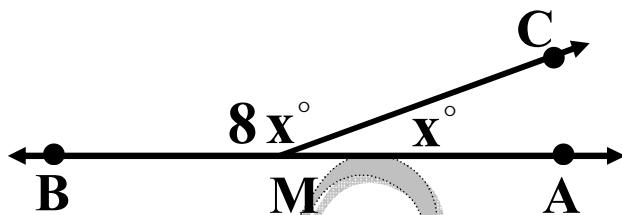
then  $m(\angle CFB) = \dots$



Test (8)**Complete:**

1) In the opposite figure:

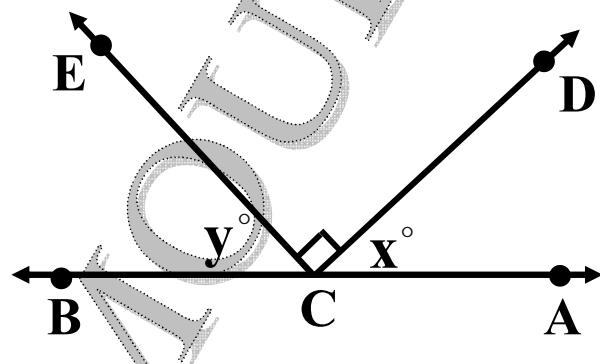
$$\text{If } M \in \overleftrightarrow{AB}, \text{ then } x = \dots\dots$$



2) In the opposite figure:

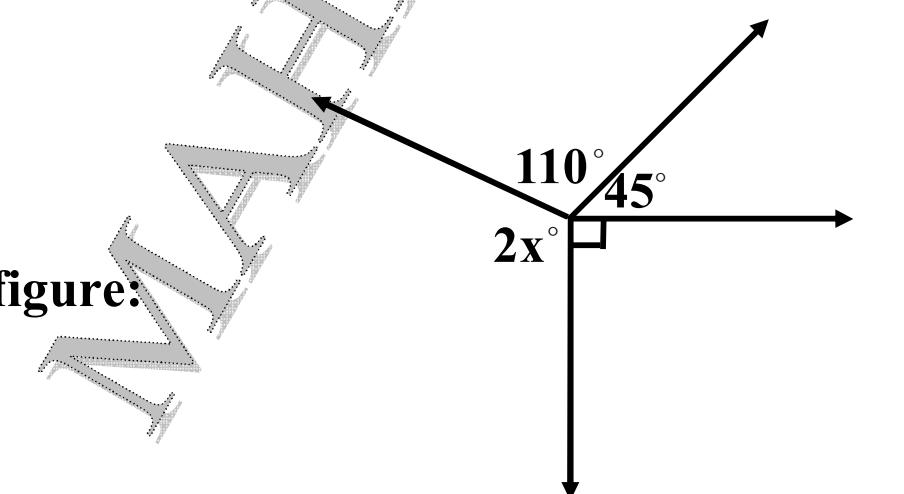
$$\text{If } C \in \overleftrightarrow{AB}$$

$$\text{then } x^\circ + y^\circ = \dots\dots$$



3) In the opposite figure:

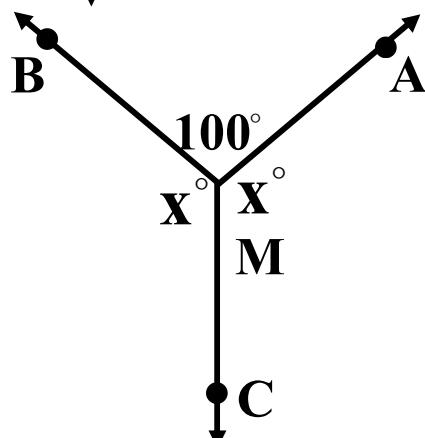
$$x = \dots\dots$$



4) In the opposite figure:

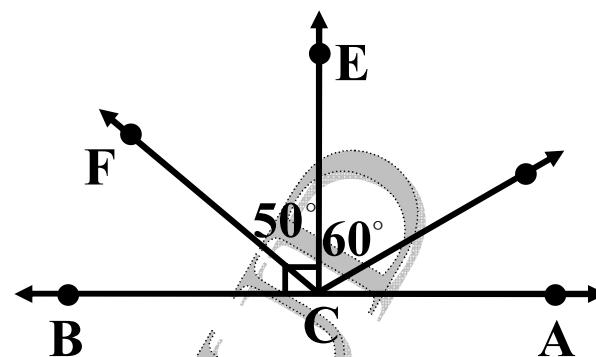
$$\text{If } m(\angle AMB) = 100^\circ$$

$$\text{, then } x = \dots\dots$$



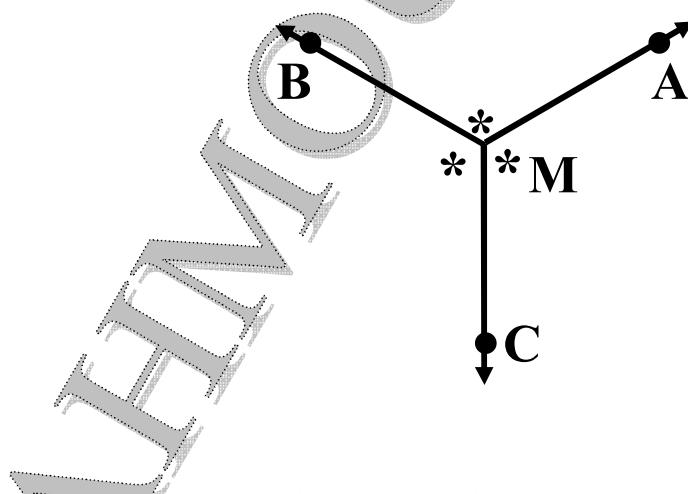
Test (9)**Complete:**

- 1) The number of obtuse angle in the opposite figure is .....



- 2) In the opposite figure:

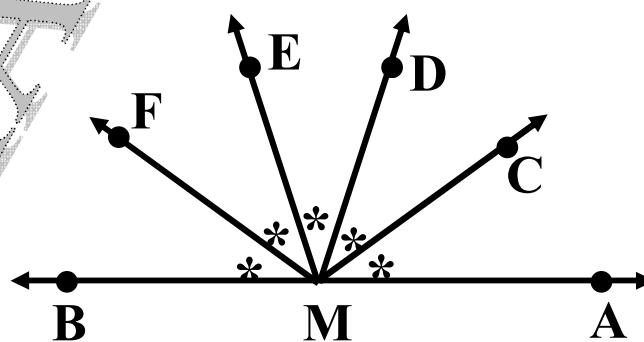
$$m(\angle AMC) = \dots \text{ } ^\circ$$



- 3) In the opposite figure:

If  $M \in \overleftrightarrow{AB}$ , then

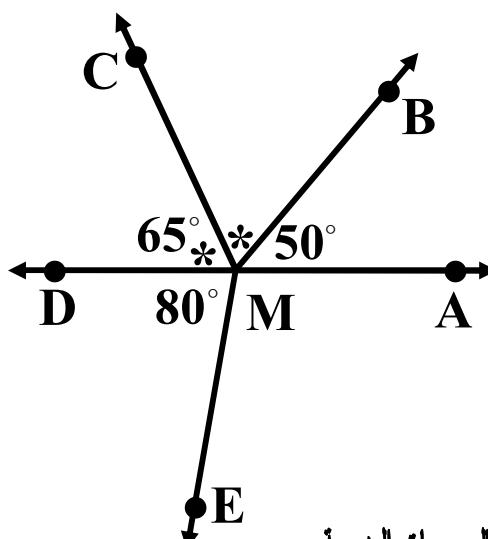
$$m(\angle AMC) = \dots \text{ } ^\circ$$



- 4) Find the measure of the required angle:

If  $\overrightarrow{MC}$  bisects  $\angle BMD$

$$\text{then } m(\angleAME) = \dots \text{ } ^\circ$$

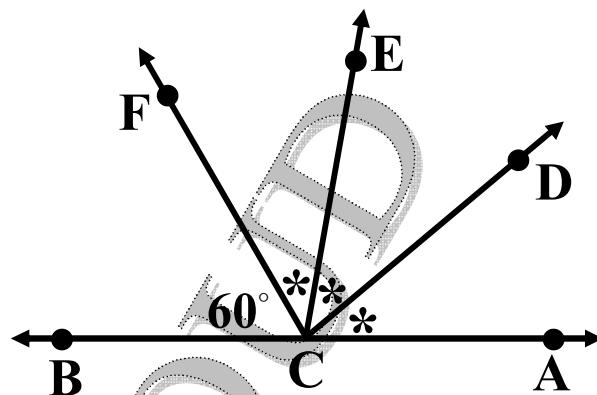


**Test (10)****Complete:**

- 1) Find the measure of the required angle:

If  $C \in \overleftrightarrow{AB}$  then

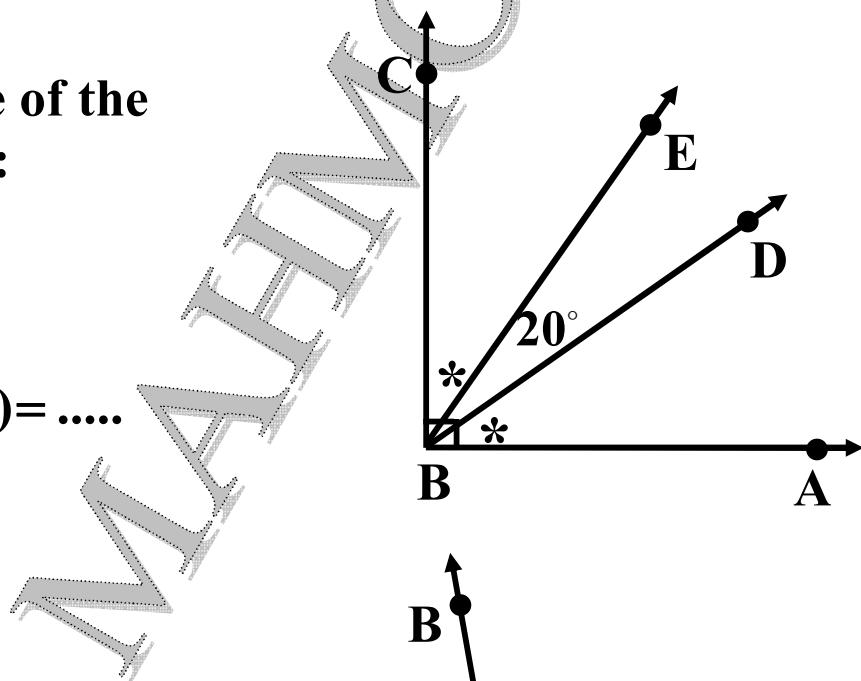
$$m(\angle DCB) = \dots\dots$$



- 2) Find the measure of the required angle:

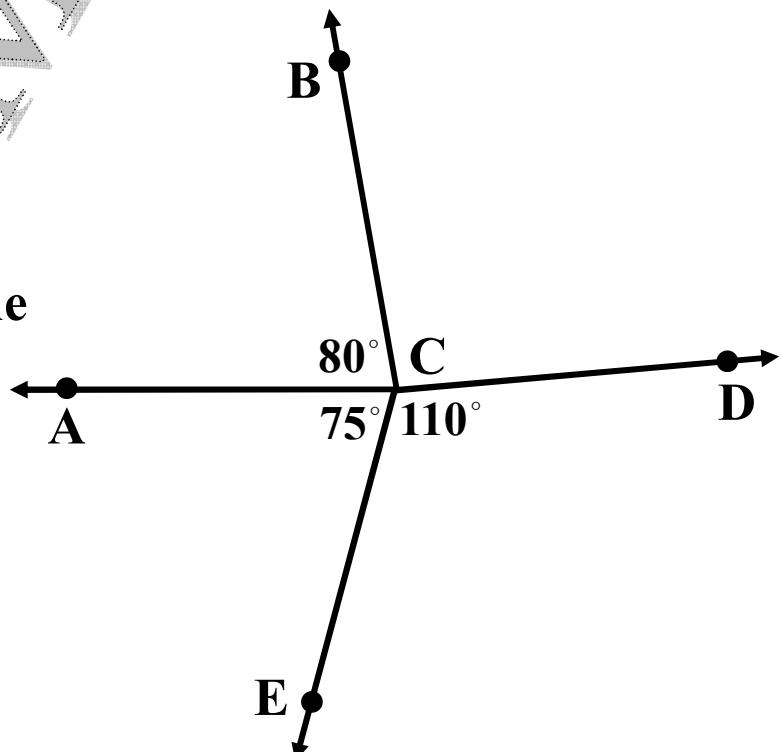
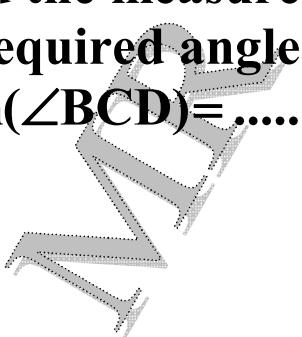
If  $\overrightarrow{BA} \perp \overrightarrow{BC}$

$$\text{then } m(\angle ABD) = \dots\dots$$



- 3) Find the measure of the required angle

$$m(\angle BCD) = \dots\dots$$



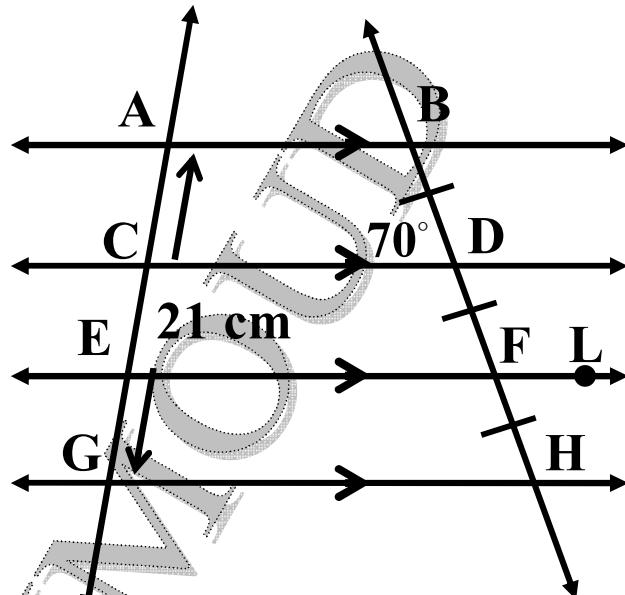
**Test (11)****[1] In the opposite figure:**

$$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD} \parallel \overleftrightarrow{EF} \parallel \overleftrightarrow{GH}$$

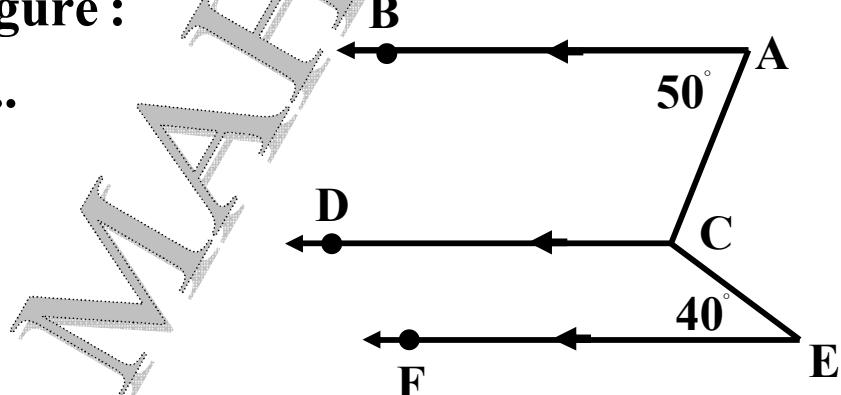
$$, AG = 21 \text{ cm}, m(\angle BDC) = 70^\circ$$

**Find:**

- 1) The length of  $\overline{AE}$
- 2)  $m(\angle ABD)$
- 3)  $m(\angle HFL)$

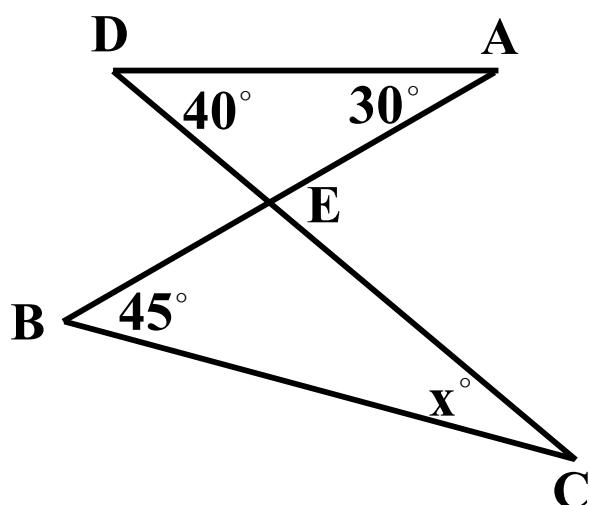
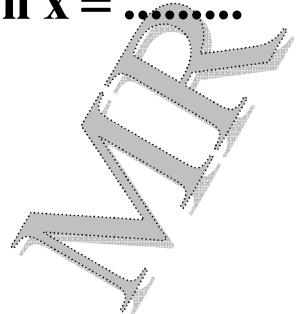
**[2] Complete:****1) In the opposite figure :**

$$m(\angle ACE) = \dots\dots\dots$$

**2) In the opposite figure :**

If  $\overline{AB} \cap \overline{CD} = \{E\}$

$$\text{then } x = \dots\dots\dots$$



**Test (12)**

**[1]** In each of the following figures , show if the two triangles are congruent or not if they are congruent name the case of congruence.

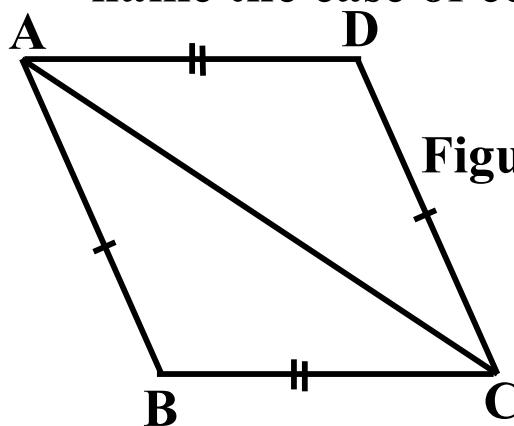


Figure (1)

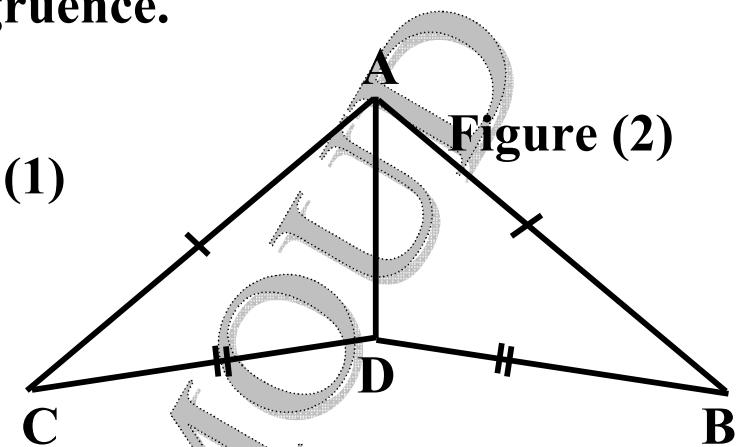


Figure (2)

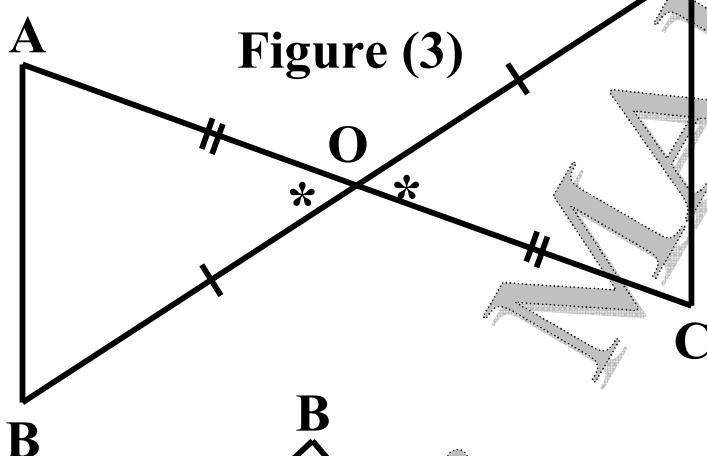


Figure (3)

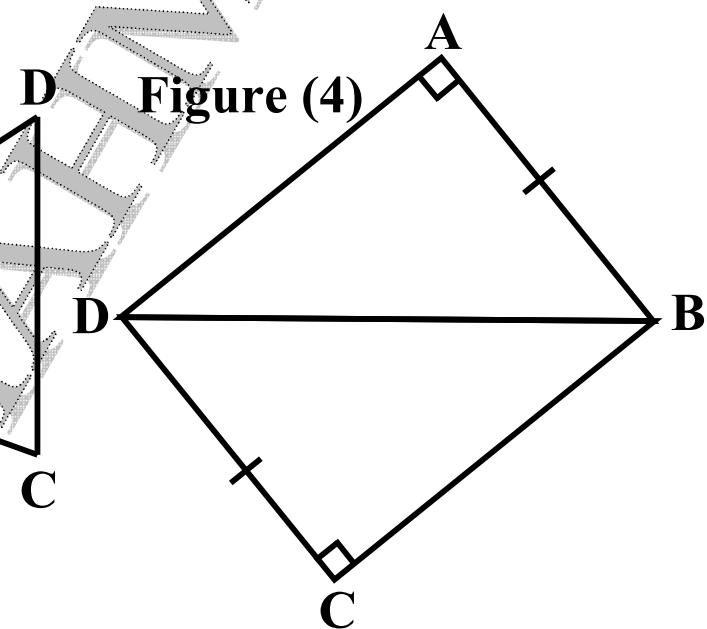


Figure (4)

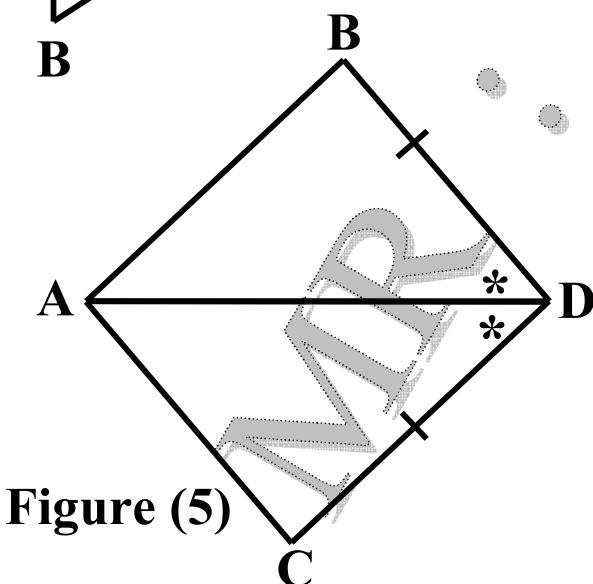


Figure (5)

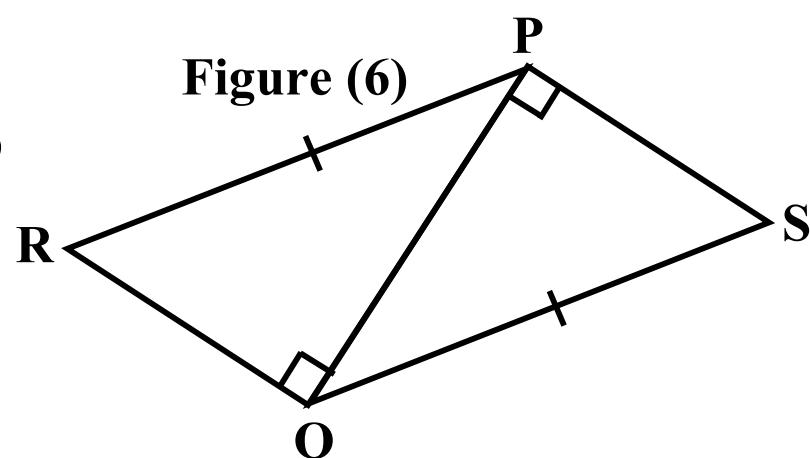
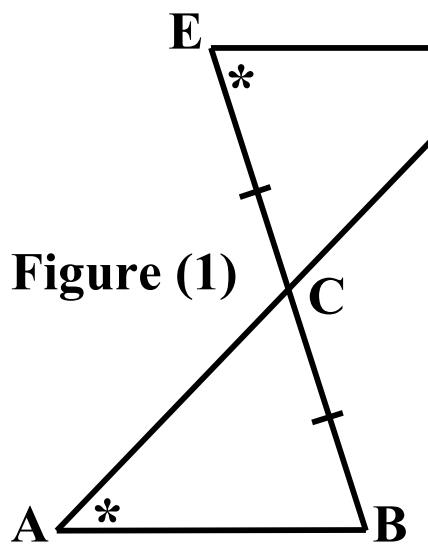
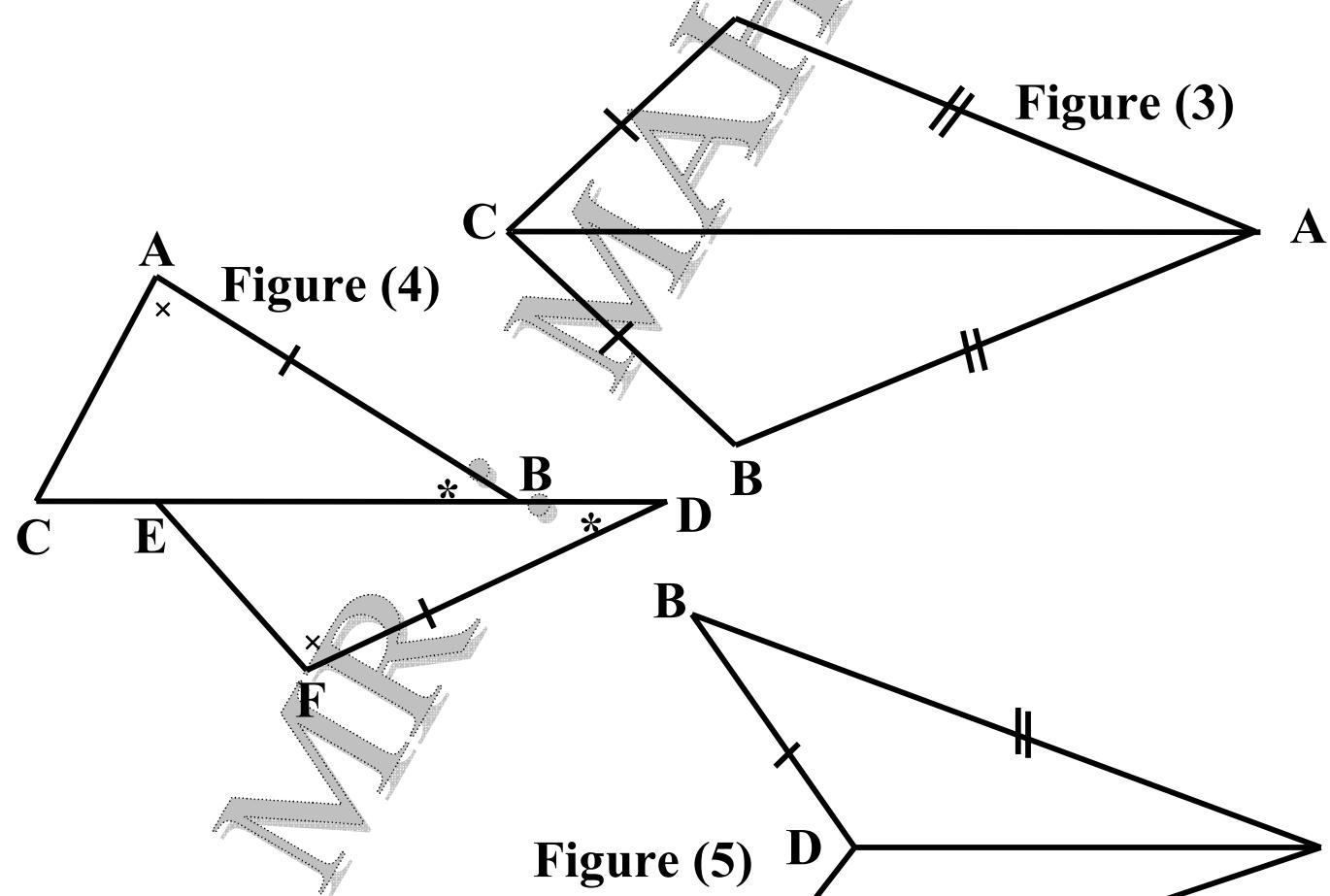
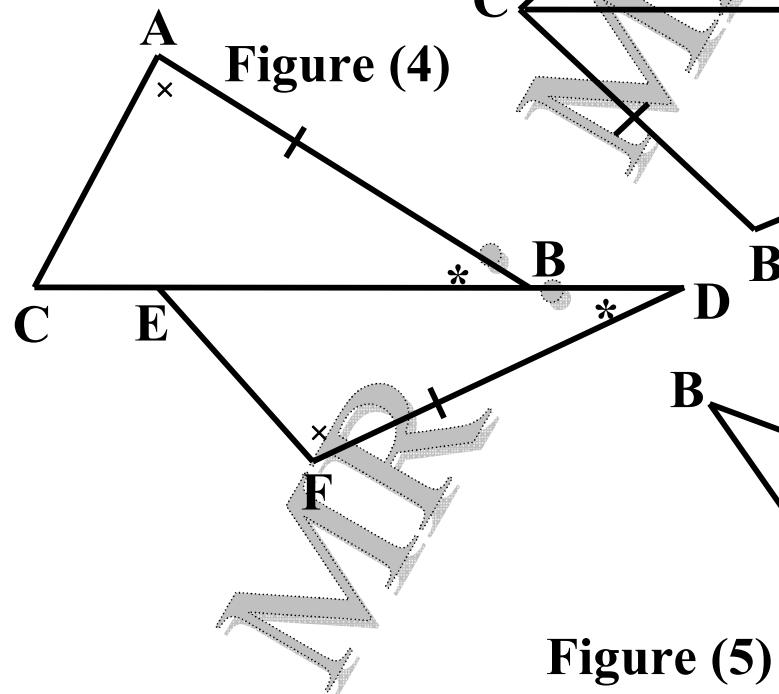


Figure (6)

**Test (13)**

**[1]** In each of the following figures , show if the two triangles are congruent or not if they are congruent name the case of congruence.

**Figure (1)****Figure (2)****Figure (3)****Figure (4)****Figure (5)**

65  
C

## [1] Complete:

1) The sum of measures of the accumulative angles at a point = .....

2) The angle whose measure is  $72^\circ$  complements the angle whose measure is .....

3) If  $\Delta ABC \cong \Delta XYZ$  and  $m(\angle X) = 50^\circ$ ,  $m(\angle B) = 60^\circ$  then  $m(\angle Z) = .....$

4) The diagonal of the rectangle divides its surface into two ..... triangles.

7) If  $m(\angle A) = 150^\circ$ , then  $m(\text{reflex } \angle A) = .....$

9) If a line segment is extended from one side without limit, the produced figure is .....

10) If  $\angle A$  supplements  $\angle B$ ,  $\angle A \equiv \angle B$ , then  $m(\angle B) = .....$

11) The measure of the straight angle = .....

13) If one of the two supplement angles is acute then the other is ..... angle.

14) The two triangles are congruent if two sides and .... in one of them are congruent to their corresponding elements in the other.

16) ..... < the measure of the obtuse angle < .....

17) If  $\Delta XYZ$  is right-angled at X,  $XY = 12 \text{ cm}$ ,  $XZ = 9 \text{ cm}$ .  
then  $(YZ)^2 = \dots \text{cm}^2$ .

19) If  $\Delta ABC \cong \Delta XYZ$  then  $BC = \dots$

20) If  $\angle A$  supplements  $\angle B$ , and  $m(\angle A) = 2 m(\angle B)$ ,  
then  $m(\angle B) = \dots$

26) If ABC is a triangle in which  $AB = 5 \text{ cm}$ ,  $BC = 12 \text{ cm}$  and  $AC = 13 \text{ cm}$  then  $m(\angle \dots) = 90^\circ$

27) The two right-angled triangles are congruent if .... in one of them are congruent with their corresponding elements in the other triangle.

31) In the right-angled triangle , the area of the square

set up the hypotenuse equals .....

35) The two adjacent angles formed by intersecting a

straight line and a ray whose start point lies on the straight line are .....

38) A rectangle of length 4 cm. and width 3 cm , then the

area of the square set its diagonal equals .....  $\text{cm}^2$

### Solution:

1)  $360^\circ$

2)  $90^\circ - 72^\circ = 18^\circ$

3)  $m(\angle C) = 180^\circ - (50^\circ + 60^\circ) = 70^\circ \therefore m(\angle Z) = m(\angle C) = 70^\circ$

4) Congruent

7)  $360^\circ - 150^\circ = 210^\circ$

9) Straightline

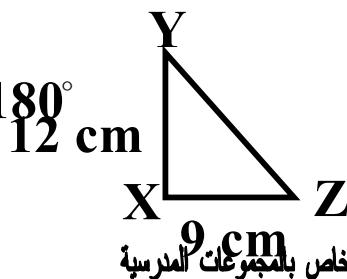
10)  $180^\circ \div 2 = 90^\circ$

11)  $180^\circ$

13) obtuse

14) The included angle

16)  $90^\circ < \text{the measure of the obtuse angle} < 180^\circ$



خالص بالمجموعات المدرسية

$$17) (YZ)^2 = 12^2 + 9^2 = 225$$

$$19) YZ$$

$$20) 180^\circ \div 3 = 60^\circ \quad \therefore m(\angle B) = 60^\circ$$

$$26) AC^2 = 13^2 = 169 , AB^2 + BC^2 = 13^2 + 5^2 = 169$$

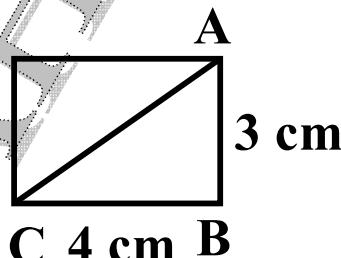
$\therefore AC^2 = AB^2 + BC^2 \quad \therefore \angle B$  is right

27) The hypotenuse and one side

31) The sum of the squares described on the other two sides

35) Supplementary

$$38) 3^2 + 4^2 = 25$$



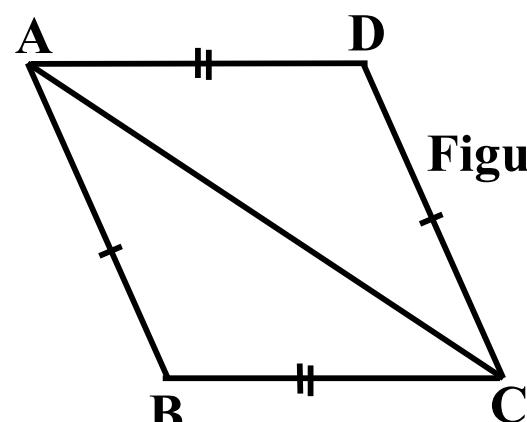
**[1]** In each of the following figures , show if the two triangles are congruent or not if they are congruent name the case of congruence.

### Figure (1)

$$\Delta ADC \cong \Delta CAB$$

where :

- 1)  $AD = BC$
- 2)  $\overline{AC}$  is a common side
- 3)  $DC = AB$



**Figure (2)**

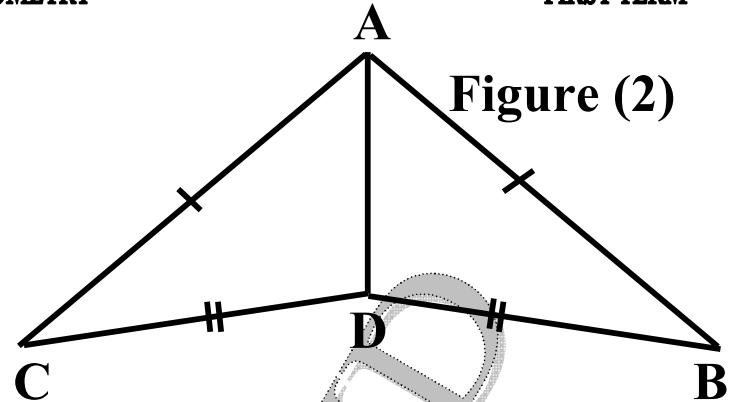
$$\Delta ADC \cong \Delta ADB$$

where :

$$1) AC = AB$$

2)  $\overline{AD}$  is a common side

$$3) DC = DB$$

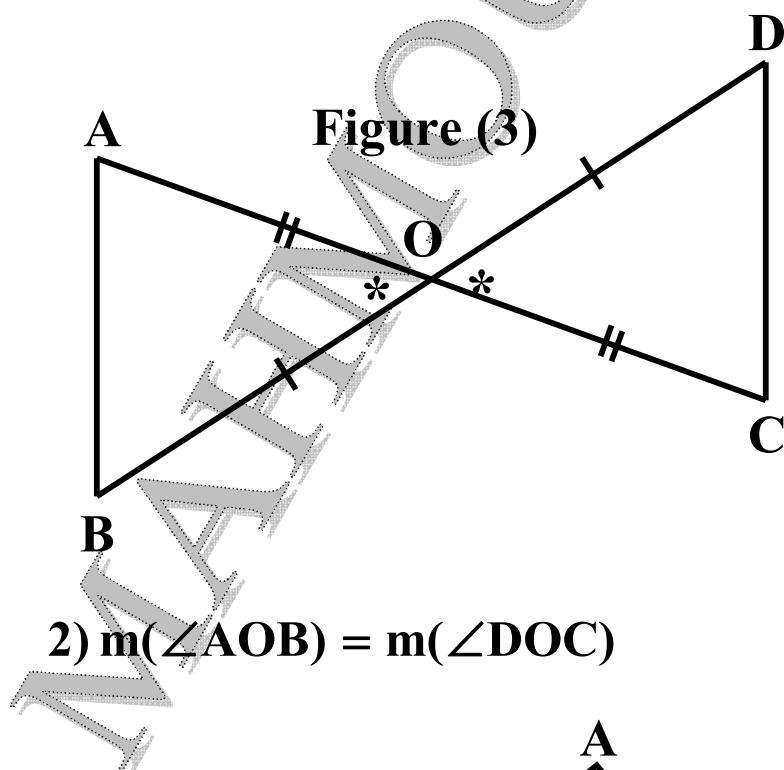
**Figure (3)**

$$\Delta AOB \cong \Delta COD$$

where :

$$1) AO = CO$$

$$3) OB = OD$$

**Figure (4)**

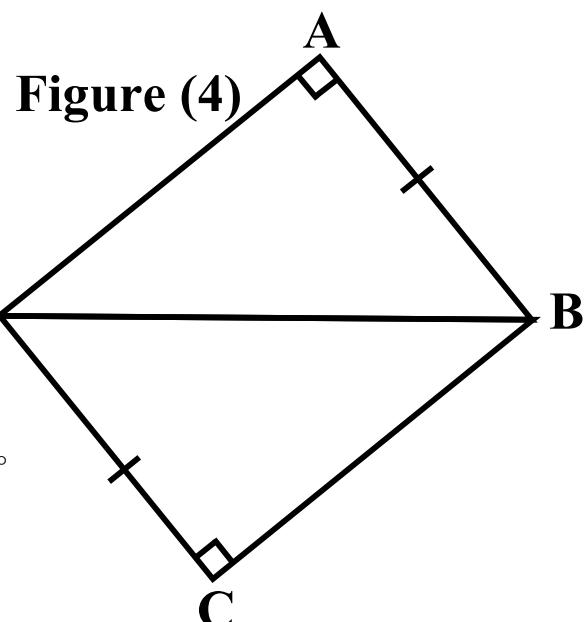
$$\Delta ADB \cong \Delta CBD$$

where :

$$1) AB = DC$$

2)  $\overline{BD}$  is a common side

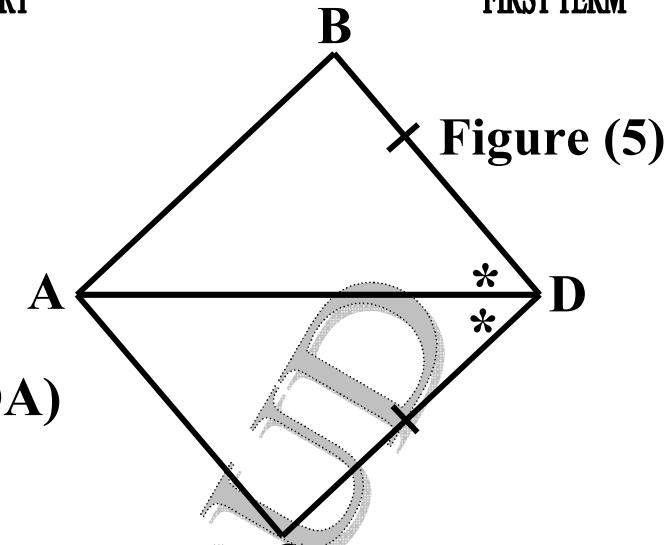
$$3) m(\angle BAD) = m(\angle BCD) = 90^\circ$$

**Figure (5)**

$$\Delta ADB \cong \Delta ADC$$

where :

- 1)  $DB = DC$
- 2)  $\overline{BA}$  is a common side
- 3)  $m(\angle BDA) = m(\angle CDA)$

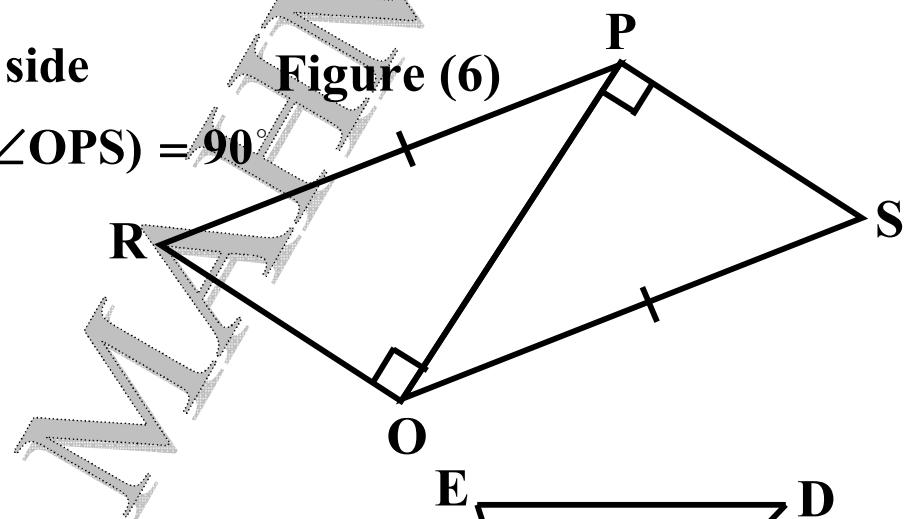


### Figure (6)

$$\Delta POR \cong \Delta OPS$$

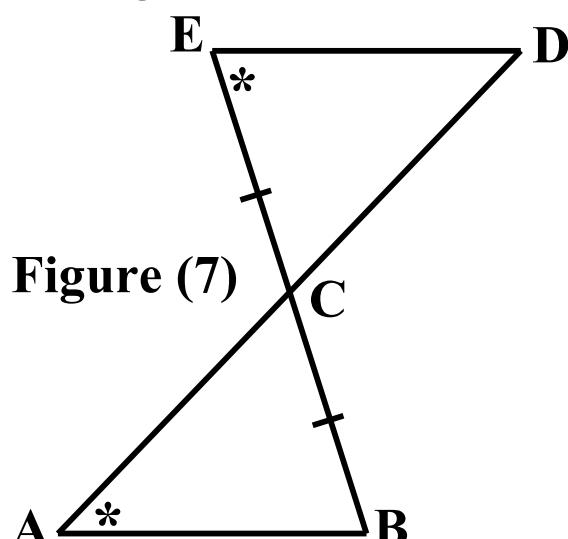
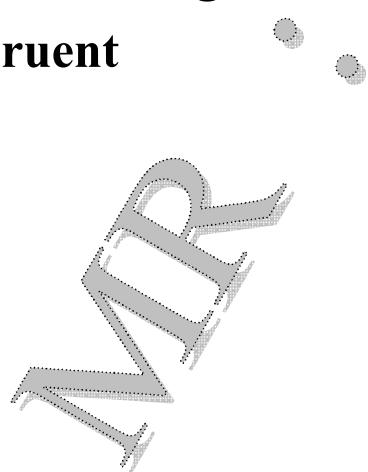
where :

- 1)  $PR = OS$
- 2)  $\overline{PO}$  is a common side
- 3)  $m(\angle POR) = m(\angle OPS) = 90^\circ$



### Figure (7)

The two triangles are not congruent

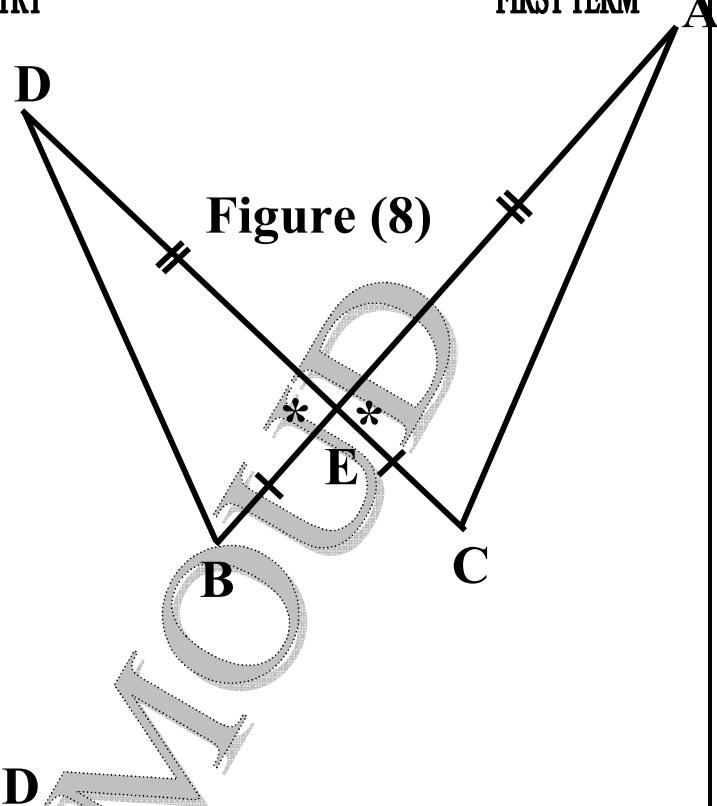


**Figure (8)**

$$\Delta DEB \cong \Delta AEC$$

where :

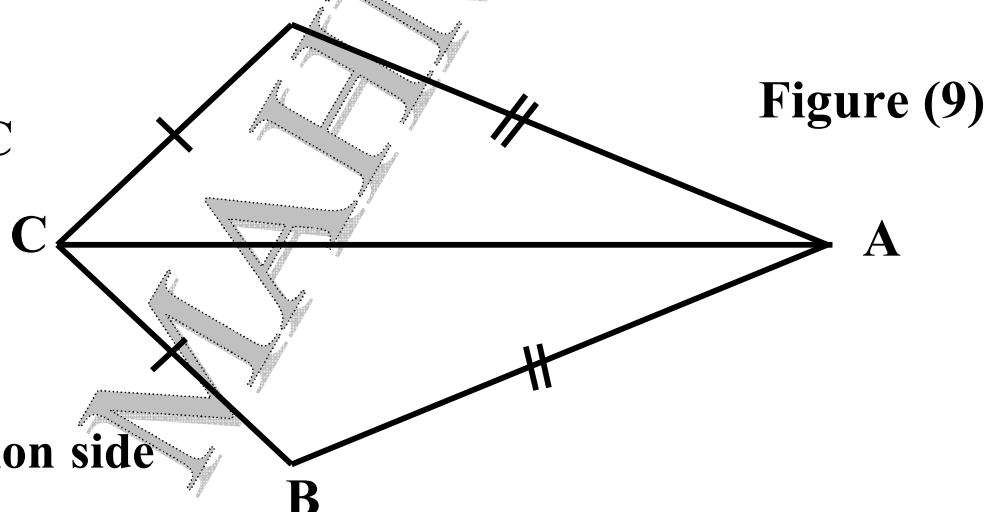
- 1)  $DE = AE$
- 2)  $BE = CE$
- 3)  $m(\angle DEB) = m(\angle AEC)$

**Figure (9)**

$$\Delta ADC \cong \Delta ABC$$

where :

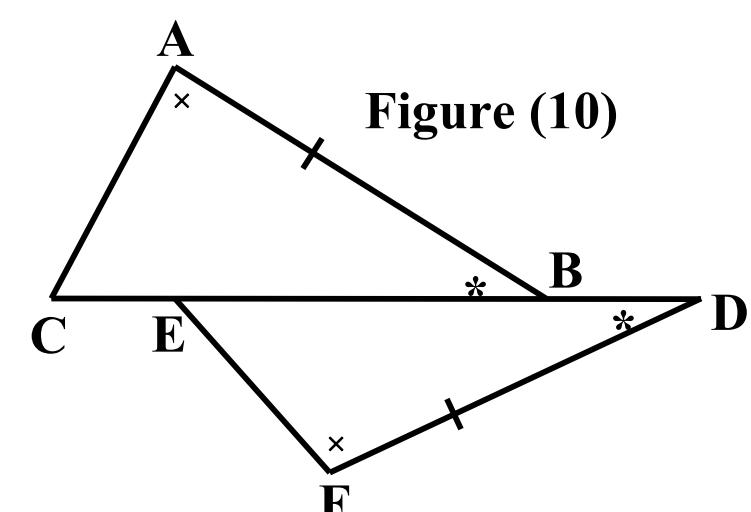
- 1)  $AD = AB$
- 2)  $DC = BC$
- 3)  $\overline{AC}$  is a common side

**Figure (10)**

$$\Delta FDE \cong \Delta ABC$$

where :

- 1)  $FD = AB$
- 2)  $m(\angle FDE) = m(\angle ABC)$
- 3)  $m(\angle DFD) = m(\angle BAC)$

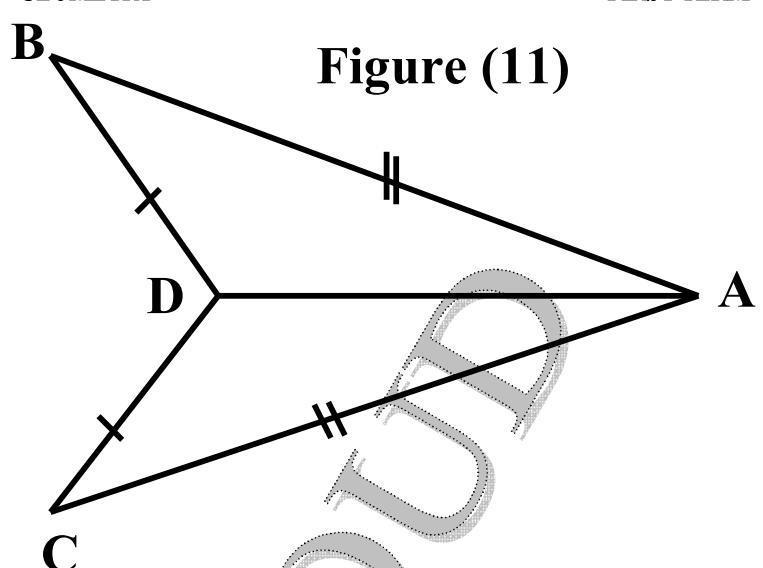


**Figure (11)**

$$\Delta ABD \cong \Delta ACD$$

where :

- 1)  $AB = AC$
- 2)  $BD = CD$
- 3)  $\overline{AD}$  is a common side



MAP

MATHMOTU

Geom.

Sheet (1)

[1] Mention the type of angle whose measure is as following :

- 1)  $57^\circ$       2)  $117^\circ$       3)  $90^\circ$   
4)  $180^\circ$       3)  $43\frac{1}{2}$       6)  $89^\circ 59' 60''$       7)  $179^\circ 62'$

[2] Complete :

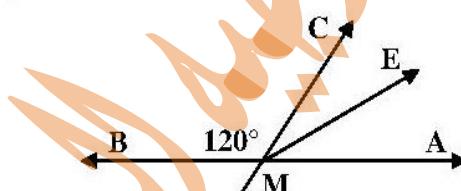
- 1) The angle is .....  
2) The measure of straight angle .....  
3) The measure of zero angle .....  
4) The measure of right angle .....  
5) The measure of acute angle is less than ..... and more than .....  
6) The measure of obtuse angle is less than more than .....  
7) The two complement angles are two angles whose sum of their measure is .....  
8) The two supplement angles are the two angles whose sum of their measure is .....  
9) The two adjacent angles formed by straight line and ray with same stating point are .....  
10) If the two outer sides of two adjacent angles are perpendicular , then these two adjacent angles are .....  
11) If the two outer sides of two adjacent angles are on the same straight line , then these adjacent angles are .....  
12) The measure of angle which complement with  $48^\circ$  is .....  
13) The measure of angle which complement with  $90^\circ$  is .....  
14) The measure of angle which complement with  $60^\circ \frac{1}{4}$  is .....  
15) Measure of angle which supplementary with  $90^\circ$  is .....angle .  
16) Measure of angle which supplementary with  $180^\circ$  is .....angle .

- 17) Measure of angle which supplementary with  $48^\circ$  .
- 18) If two straight lines intersect then the measure of each two vertically opposite angle are .....
- 19) The sum of measure of accumulative angles at point .....
- 20) Angle bisector is .....
- 21) If  $m(\angle A) = 80$  then ( reflex  $\angle A$ ) = ..... $^\circ$

**22) In opposite figure :**

- a) M is the point intersection of  $\overleftrightarrow{AB}$  and  $\overleftrightarrow{CD}$  , ME bisects  $\angle AMC$  and  $m(\angle BMC) = 120^\circ$  . Find :

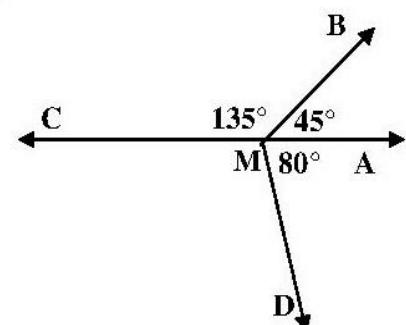
$$m(\angle AMC) , m(\angle AMD) , m(\angle AME)$$



**b) In the figure :**

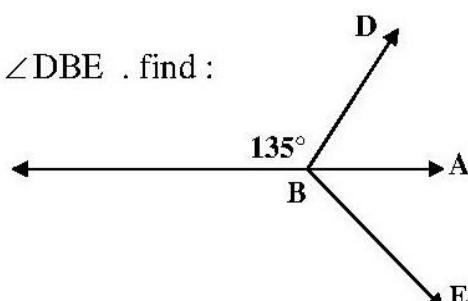
$$1) m(\angle CMD) = .....^\circ$$

2) .....and .....lie on the same straight line .



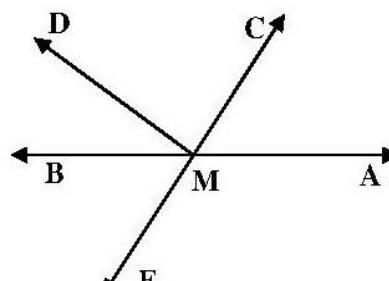
- c) If  $B \in AC$  ,  $m(\angle DBC) = 135^\circ$  and  $\overrightarrow{BA}$  bisects  $\angle DBE$  . find :

$$m(\angle ABD) , m(\angle DBE) , m(\angle CBE)$$



- d) If  $\overleftrightarrow{AB} \cap \overleftrightarrow{CE} = \{M\}$  ,  $MD \perp CE$  and  $\overrightarrow{MB}$  bisects  $\angle DME$  . Find :

$$m(\angle BME) , m(\angle DME) , m(\angle AMC) , m(\angle AME)$$



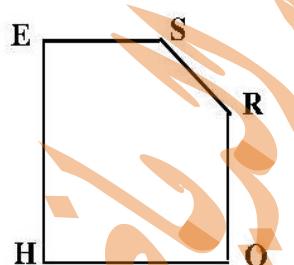
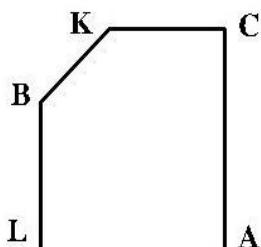
## Sheet (2)

### **[1] Complete :**

- 1) The two line segment are congruent if .....
- 2) The two angles are congruent if .....
- 3) The two square are congruent if .....
- 4) The two rectangle are congruent if .....

### **[2] In the opposite figure :**

The two pentagons shown are congruent



### **Complete :**

- 1) B correspond to .....
- 2) The polygon BLACK is congruent the polygon .....
- 3) KB = ..... cm.
- 4)  $M(\angle E) = m(\angle \dots)$
- 5) CA = ..... cm
- 6)  $M(\angle A) = m(\dots)$

### **[3] In the opposite figure :**

If  $C \in BD$  ,  $m(AFC) = 110^\circ$  ,  $BC = 5$  cm and polygon ABCF  $\equiv$  the polygon EDCF  
 $ED = 8$  cm ,  $EF = 4$  cm .

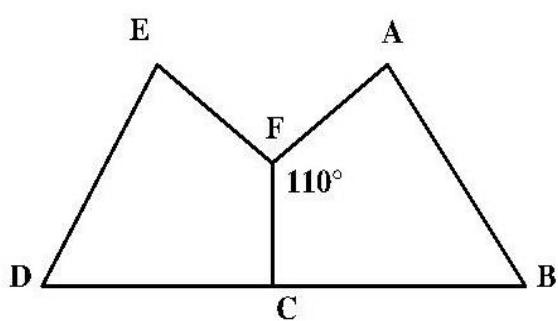
Complete :

$M(EFC) = \dots$

$DC = \dots$  cm

$AB = \dots$

$AF = \dots$



### Sheet (3)

- 1) Draw the line segment whose length 7 cm. then divid it into two equal parts in length using the compass and the an scaled ruler .

- 2) Draw  $\angle ABC$  where  $m(\angle B) = 80^\circ$  using the ruler and compasses bisect  $\angle B$  by  $\overrightarrow{BD}$

- 3) Use the ruler and compasses to draw the equilateral  $\triangle ABC$  of side 6 cm . Draw  $\overline{AD} \perp \overrightarrow{BC}$  where  $\overline{AD} \cap \overrightarrow{BC} = \{D\}$  . what the length of  $\overline{AD}$  .

- 4) Draw  $\angle XYZ$  whose measure  $70^\circ$  use ruler and draw congruent equal to it .

- 5) Using the protractor , draw  $\angle ABC$  with measure  $70^\circ$  and on the other side of BA , draw using ruler and compasses draw  $\overrightarrow{AE} // \overrightarrow{BC}$  .

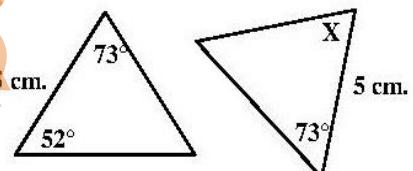
### Sheet (4)

#### [1] Complete the following :

- 1) Any two triangles are congruent if two sides .....
- 2) Any two triangles are congruent if two angles and ..... in one of the triangles are congruent to their corresponding element in the other .
- 3) Any two triangles are congruent if each ..... is congruent to its corresponding side in the other triangle .
- 4) Any two right – angled triangles are congruent if .....
- 5) The diagonal of the rectangle divides its surface into two ..... triangles .
- 6) If  $\triangle ABC \cong \triangle XYZ$  , then  $AB = \dots$  and  $m(\angle Z) = m(\angle \dots)$

#### [2] In the opposite figure :

These triangles are congruent , then  $X = \dots^\circ$



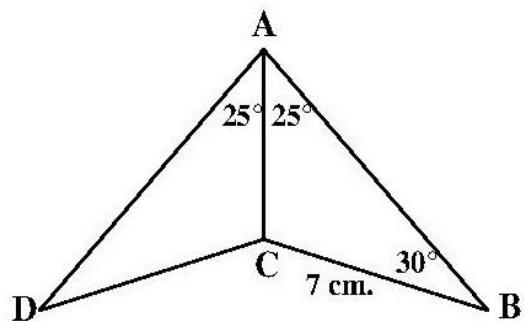
#### [3] In the opposite figure :

If  $AB = AD$  ,  $BC = 7 \text{ cm}$  ,  $m(\angle BAC) = m(\angle DAC) = 25^\circ$  and  $m(\angle B) = 30^\circ$

#### Complete the following :

If  $\triangle ACB \cong \triangle ACD$

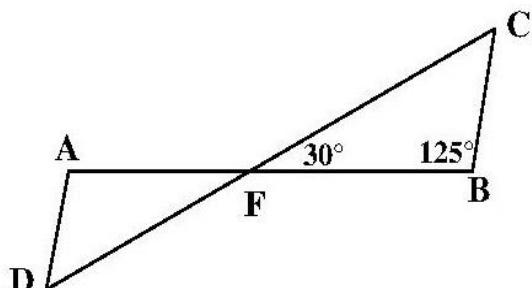
- 1)  $m(\angle D) = \dots^\circ$
- 2)  $CD = \dots \text{ cm.}$
- 3)  $m(\angle ACD) = \dots^\circ$



#### [4] In the opposite = {F} , $FA = FB$ , $CF = FD$ ,

$m(\angle CFB) = 30^\circ$  and  $m(\angle B) = 125^\circ$  ,

Then  $m(\angle D) = \dots^\circ$

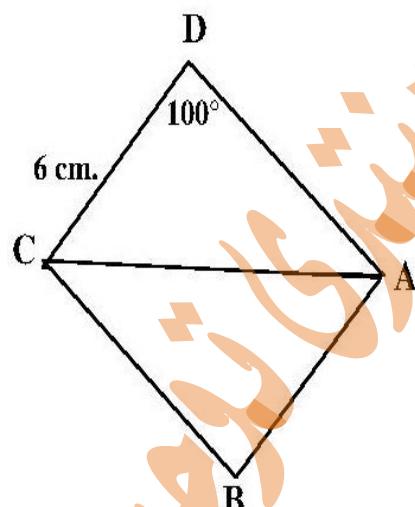


**[5] In the opposite figure :**

If  $\overleftrightarrow{AC}$  bisects  $\angle DCB$ ,  $\angle DAB$ ,  $m(\angle D) = 100^\circ$

And  $DC = 6 \text{ cm}$ . complete the following :

- 1)  $\Delta ADC \cong \Delta \dots$
- 2)  $m(\angle B) = \dots$
- 3)  $BC = \dots \text{ cm}$ .

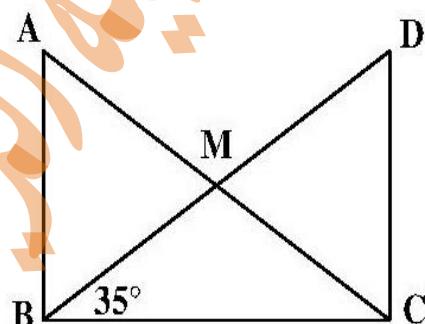


**[6] In the opposite figure :**

$AB = CD$ ,  $m(\angle DBC) = 35^\circ$ ,

$\overline{AB} \perp \overline{BC}$  and  $\overline{DC} \perp \overline{BC}$ ,

Then  $m(\angle BMC) = \dots$



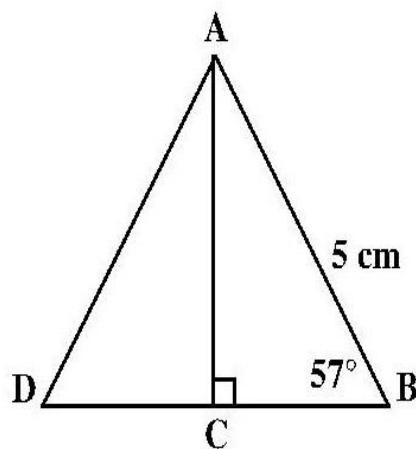
**[7] In the opposite figure :**

C is the midpoint of  $\overline{BD}$ ,  $\overline{AC} \perp \overline{BD}$ ,

$AB = 5 \text{ cm}$ , and  $m(\angle B) = 57^\circ$

Find :

- 1) The length of  $\overline{AD}$
- 2)  $m(\angle DAC)$

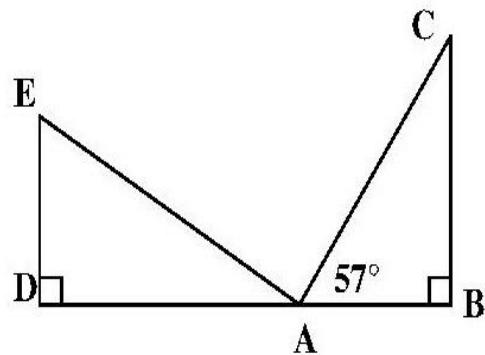


**[8] In the opposite figure :**

$BC = AD$ ,  $AC = AE$

And  $m(\angle CAB) = 57^\circ$

Find the measures of the unknown angles in  $\Delta ADE$



### Sheet (5)

**[1] Complete the following :**

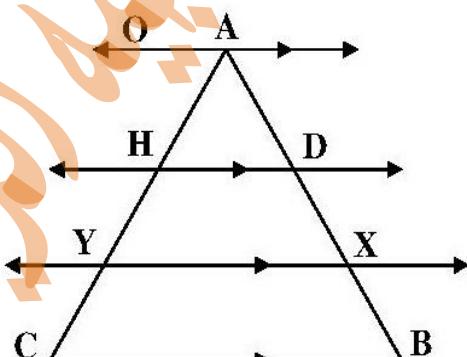
- 1) If two straight lines are parallel to a third straight line , then they are .....
- 2) If a straight line cuts two parallel straight lines , then each two corresponding angles are .....
- 3) If a straight line cuts two parallel straight lines , then each two interior angles in the same side of the transversal are .....

**[2] In the opposite figure :**

$$\overline{AO} \parallel \overline{HD} \parallel \overline{YX} \parallel \overline{CB}$$

,  $AD = DX = XB$  and  $AC = 18 \text{ cm}$ .

Find the length of  $AY$



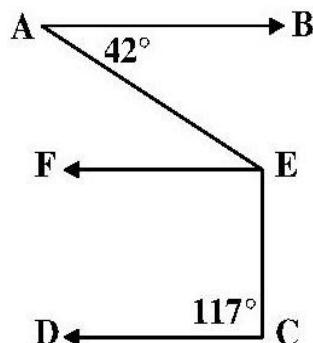
**[3] In the opposite figure :**

$$\overrightarrow{AB} \parallel \overrightarrow{CD}, \overrightarrow{EF} \parallel \overrightarrow{CD}$$

,  $m(\angle A) = 42^\circ$  and  $m(\angle C) = 117^\circ$

Determine :

$$m(\angle AEC)$$



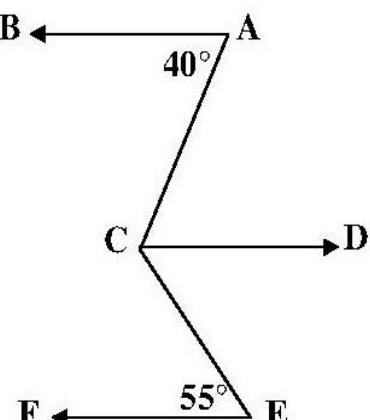
**[4] In the opposite figure :**

$$m(\angle A) = 40^\circ, m(\angle E) = 55^\circ$$

$$\overrightarrow{AB} \parallel \overrightarrow{EF} \text{ and } \overrightarrow{AB} \parallel \overrightarrow{CD}$$

Find :

$$M(\angle ACE)$$

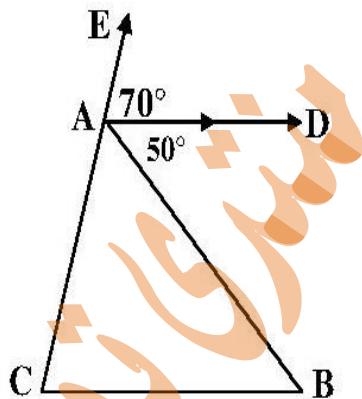


[5] In the opposite figure :

$$\overrightarrow{AD} \parallel \overrightarrow{BC}, E \in \overrightarrow{CA},$$

$$m(\angle DAE) = 70^\circ \text{ and } m(\angle DAB) = 50^\circ$$

Find the measures of the triangle ABC



[6] In the opposite figure :

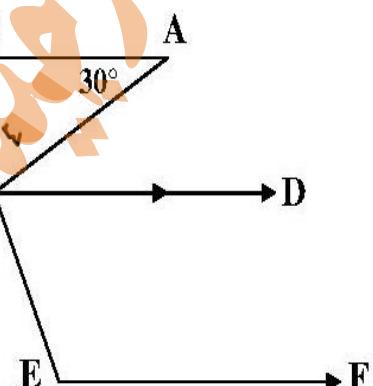
$$\overrightarrow{AB} \parallel \overrightarrow{CD} \parallel \overrightarrow{EF}, m(\angle A) = 35^\circ \text{ and}$$

$\overrightarrow{CD}$  bisects  $\angle ACE$

Find :

$$1) m(\angle DCE)$$

$$2) m(\angle CEF)$$

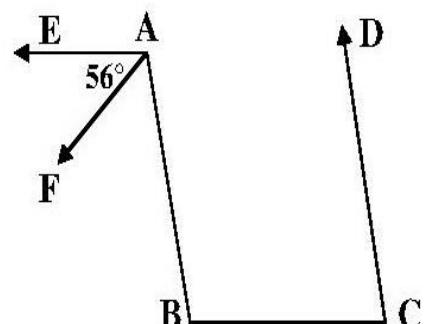


[7] In the opposite figure :

$$\overrightarrow{AE} \parallel \overrightarrow{CB}, \overrightarrow{BA} \parallel \overrightarrow{CD},$$

$$\overrightarrow{AF} \text{ bisects } \angle BAE \text{ and } m(\angle EAF) = 56^\circ$$

Find :  $m(\angle C)$



[8] In the opposite figure :

$$\overrightarrow{XL} \parallel \overrightarrow{YZ}, \overrightarrow{XY} \parallel \overrightarrow{LZ} \text{ and } m(\angle XYM) = 100^\circ$$

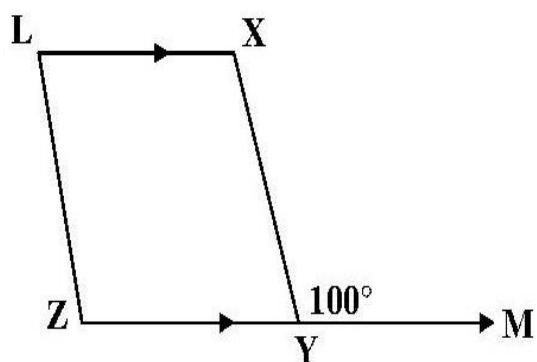
Where  $M \in \overrightarrow{ZY}$

Find :

$$1) m(\angle X)$$

$$2) m(\angle Z)$$

$$3) m(\angle L)$$



## Geometric concepts

The line segment



The straight line



The ray



[1] In the opposite figure :

A , B , C and D are points lying on one line ,  $AD \cap BE = \{B\}$

Complete each of the following by using  $\in$  ,  $\notin$  ,  $\subset$  or  $\not\subset$ :

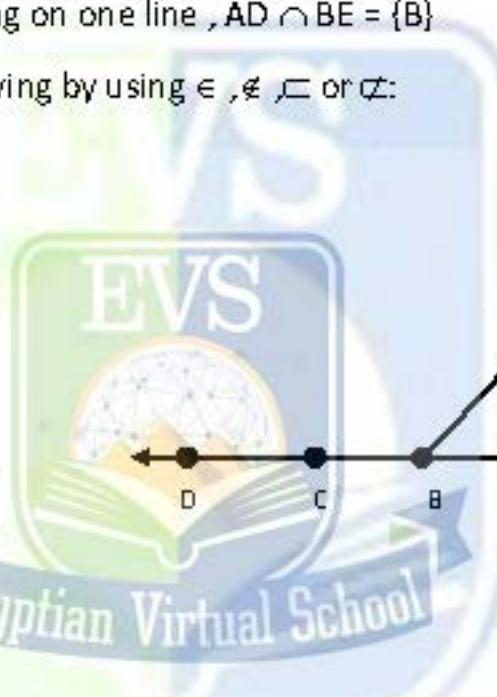
1) A  DC

2) C  AB

3) DC  AB

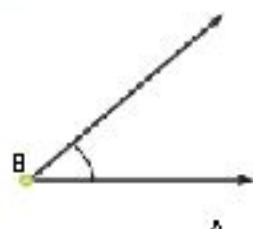
4) A   $\angle EBC$

5) AC  AD



**The angle:** is the union of two rays having the same starting point.

\*The common point of the two rays is called the **vertex** of the angle. (B)



\*Each of the two rays is called a **side** of the angle. ( $\overrightarrow{BA}$  ,  $\overrightarrow{BC}$ )

C

## \* The Types of angles

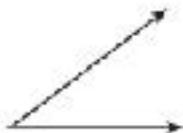
### 1) Zero angle:

- Its measure =  $0^\circ$



### 2) Acute angle:

- Its measure is more than  $0^\circ$  and less than  $90^\circ$



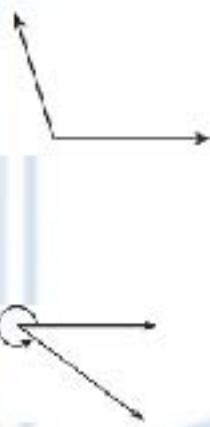
### 3) Right angle:

- Its measure =  $90^\circ$



### 4) Obtuse angle:

- Its measure is more than  $90^\circ$  and less than  $180^\circ$



### 5) Straight angle:

- Its measure =  $180^\circ$



### 6) Reflex angle:

- Its measure is more than  $180^\circ$  and less than  $360^\circ$



### [1] Mention the type of each of the following angles:

a)  $57^\circ$  .....

b)  $90^\circ$  .....

c)  $89^\circ$  .....

d)  $270^\circ$  .....

e)  $0^\circ$  .....

f)  $90\frac{1}{2}^\circ$  .....

g)  $65^\circ 15'$  .....

h)  $89^\circ 70'$  .....



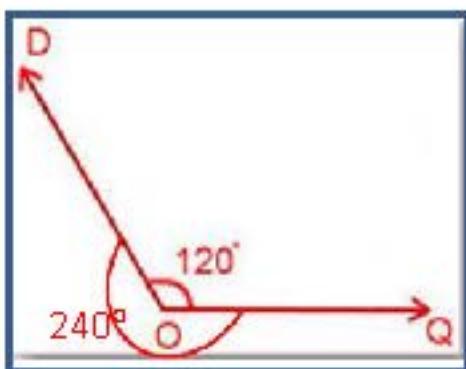


**Remark**

In the opposite figure :

$$m(\angle QOD) + m(\text{reflex } \angle QOD)$$

$$120^\circ + 240^\circ = 360^\circ$$



**[2] complete:**

- The angle whose measure is  $25^\circ$ , its type is .....
- The angle whose measure is  $179^\circ 60'$ , its type is .....
- If  $m(\angle B) = 160^\circ$ , then  $m(\text{reflex } \angle B) = \dots^\circ$

**[3] Draw the following angles :**

a)  $115^\circ$

b)  $80^\circ$

c)  $300^\circ$



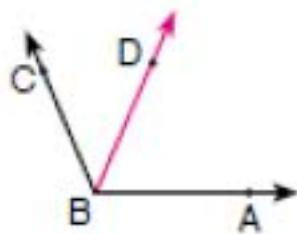
## \*Some relations between the angles

### 1) Adjacent angles:

Two angles are said to be adjacent if they have:

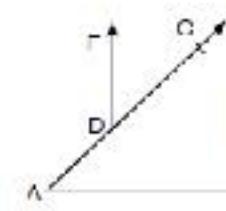
a common vertex,

a common side and the other two sides are on opposite sides of the common side.

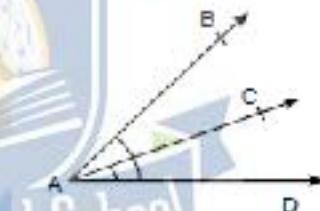


$\angle ABD$ ,  $\angle DBC$  are adjacent

[1] Which of the following angles are adjacent? (give reason)



$\angle BAC$  and  $\angle EDC$



$\angle BAC$  and  $\angle BAD$



### Complementary angles:

Two angles are said to be complementary if their sum is  $90^\circ$ .



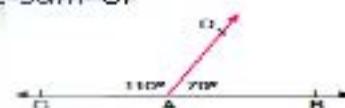
complementary angles

[2] Complete:

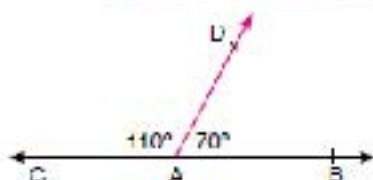
- 1) The angle of measure  $40^\circ$  complements angle of measure .....
- 2) The acute angle complements .....angle
- 3) Zero angle is complemented by .....angle

### Supplementary angles:

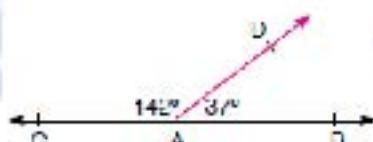
Two angles are said to be supplementary if the sum of their measure is  $180^\circ$



[3] Which of the following represent supplementary angles? Give reason.



(1)



(2)

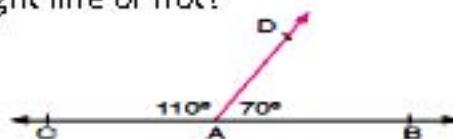
[4] Complete:

- 1) The angle of measure  $80^\circ$  supplements angle of measure .....
- 2) The obtuse angle supplements ..... angle.



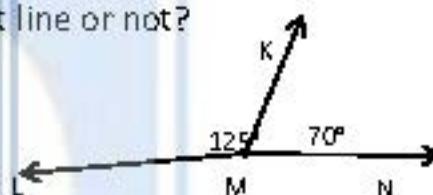
[5] In the opposite figure:

Find if:  $\angle CAD$  and  $\angle BAD$  are on the same straight line or not?



[6] In the opposite figure:

Find if:  $\angle KML$  and  $\angle KMN$  are on the same straight line or not?

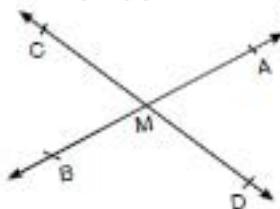


[7] If the ratio between two supplementary angles is  $7 : 11$ , then the measure of the smaller angle is : .....



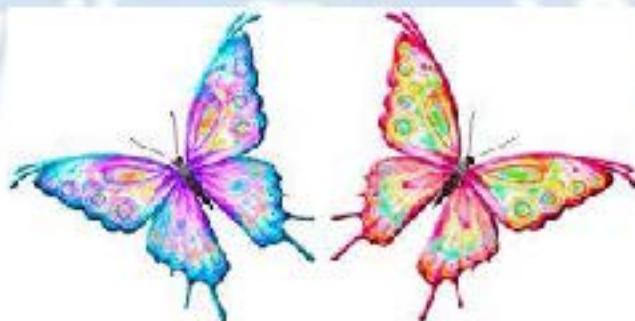
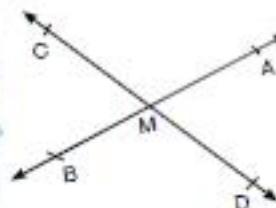
### Vertically opposite angles (V.O.A):

If two straight lines intersect, then each two vertically opposite angles are equal in measure



[1] Complete:

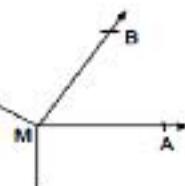
- 1) The angle whose measure is  $46^\circ$  is vertically opposite to an angle whose measure is .....
- 2) The right angle is vertically opposite angle to .....
- 3) The two angles AMD, BMC are called .....



### **Accumulative angles at a point:**

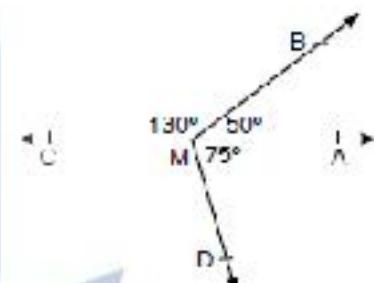
The sum of the measures of the accumulative angles at a point is  $360^\circ$

$$1) m(\angle AMB) + m(\angle BMC) + m(\angle CMD) + m(\angle DMA) = \dots$$



\*Find:

$$m(\angle CMD) = \dots^\circ$$



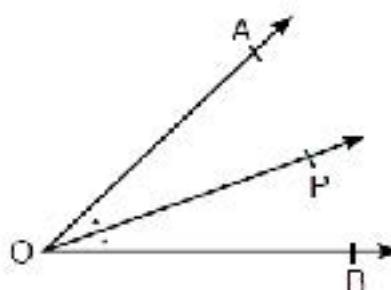
### **The angle bisector:**

It is the ray that divides the angle into two halves

(two equal angles in measure)

[1]  $\overrightarrow{OP}$  divides  $\angle AOB$  into two angles having the same measure and  $\overrightarrow{OP}$  is called the bisector of  $\angle AOB$ .

$$\text{Then } m(\angle AOP) = m(\angle BOP)$$

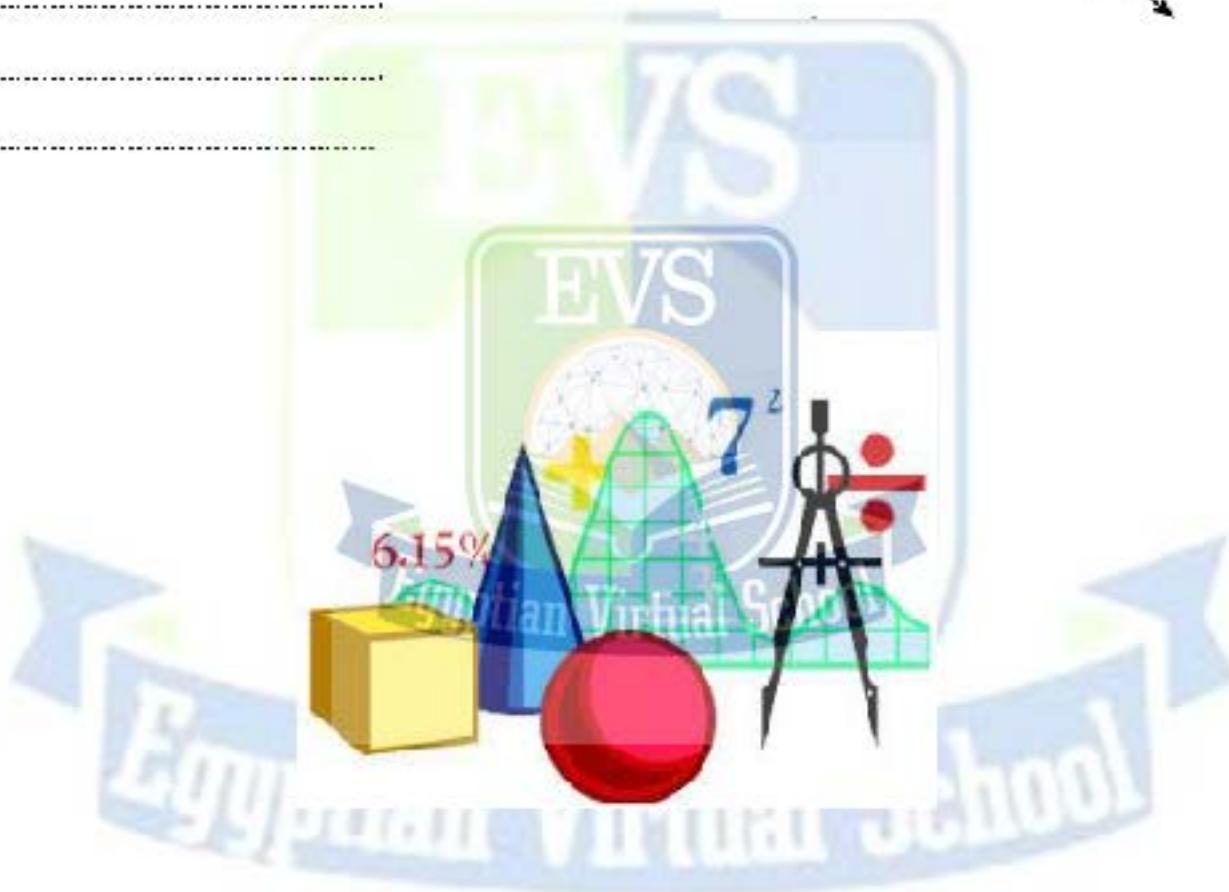
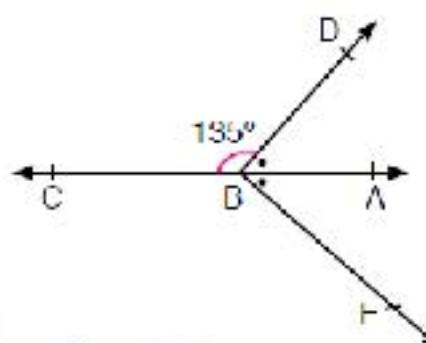


[2] In the figure opposite,

If  $B \in AC$ ,  $m(\angle DBC) = 135^\circ$  and

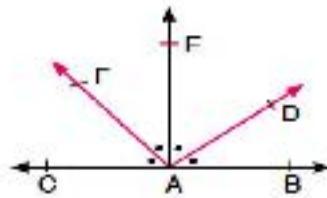
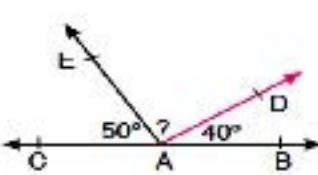
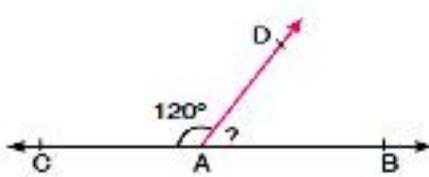
$\overrightarrow{BA}$  bisects  $\angle DBE$  find:

$m(\angle ABD)$ ,  $m(\angle DBE)$ ,  $m(\angle CBE)$



Try by yourself

[1] Find the measure of each of the following unknown angles:

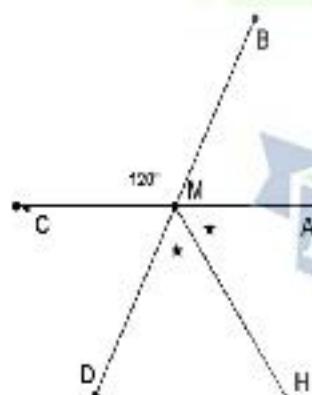


$$m(\angle BAD) = \dots \text{°}$$

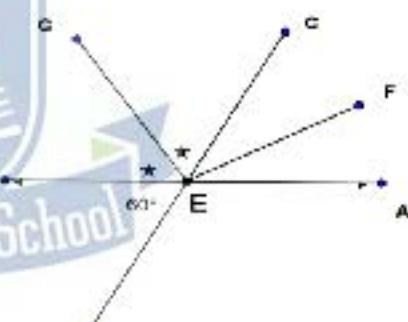
$$m(\angle DAE) = \dots \text{°}$$

$$m(\angle BAD) = \dots \text{°}$$

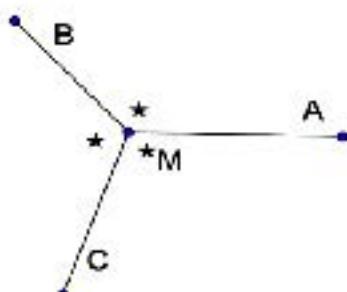
[2] In each of the following figures, find the measure of the required angles:



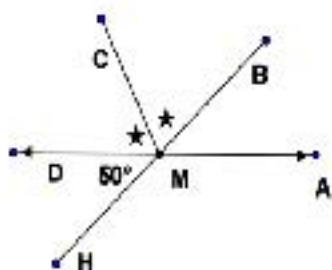
$$m(\angle HMD) = \dots \text{°}$$



$$m(\angle GEB) = \dots \text{°}$$



$$m(\angle AMC) = \dots \text{°}$$



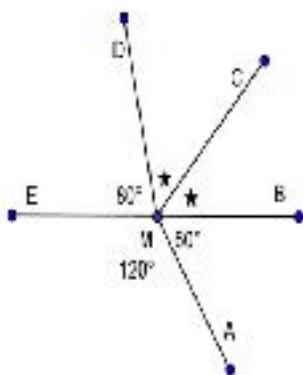
$$m(\angle AMC) = \dots \text{°}$$

**2) In the opposite figure :**

$$m(\angle A M B) = 60^\circ, m(\angle A M E) = 120^\circ,$$

$$m(\angle E M D) = 80^\circ$$

and  $\overrightarrow{MC}$  bisects  $\angle BMD$



**Find :**

$$1) m(\angle CMD)$$

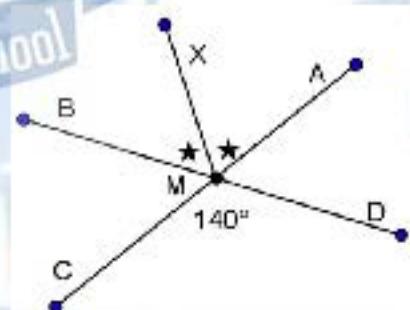
$$2) m(\angle AMC)$$

**3) In the opposite figure :**

$$\overleftrightarrow{AC} \cap \overleftrightarrow{BD} = \{M\}, MX \text{ bisects } \angle AMB$$

$$\text{and } m(\angle CMD) = 140^\circ$$

Find :  $m(\angle DMX)$

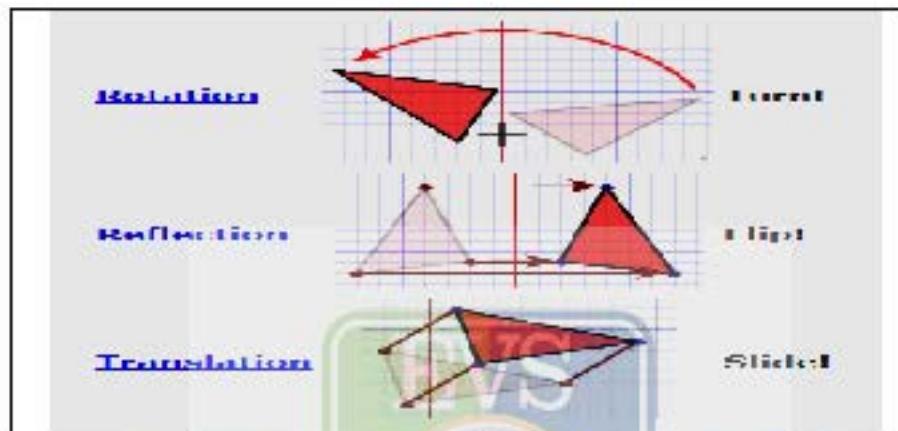




## Congruence



\*If one shape can become another using Turns, Flips or Slides, then the shapes are **Congruent**:



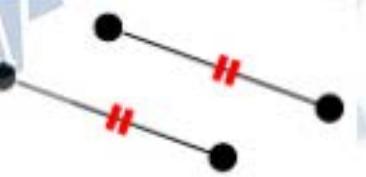
### [1] Congruence of two line segments:

\*Two line segments are congruent if they are equal in length.

Complete:

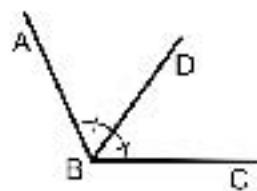
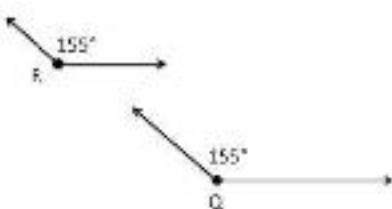
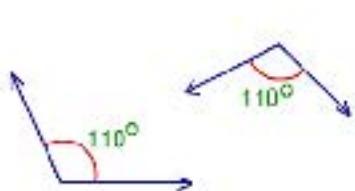
1) If  $\overline{AB} \cong \overline{CD}$ , Then  $AB \dots CD$

2) If  $\overline{AB} \cong \overline{CD}$ , Then  $AB - CD = \dots$



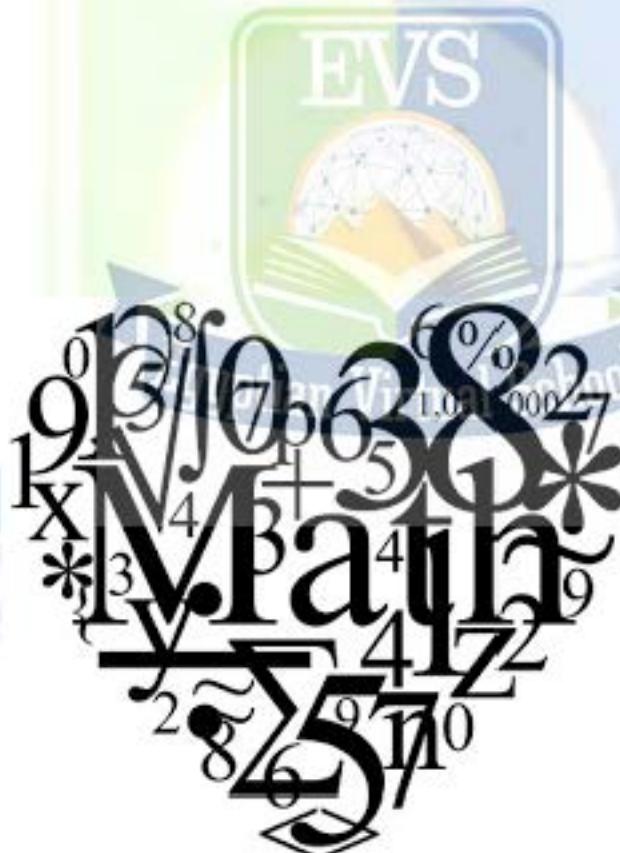
## [2] Congruence of two angles:

\*Two angles are congruent if they are equal in measure.



Complete:

- 1) If  $\angle A \cong \angle B$ , and  $m(\angle A) = 50^\circ$ , then  $m(\angle B) = \dots^\circ$
- 2) If  $\angle A$  complements  $\angle B$ , and  $\angle A \cong \angle B$ , and  $m(\angle A) = 50^\circ$ , then  $m(\angle B) = \dots^\circ$



### [3]Congruence of two polygons:

\*Two polygons are congruent if there is a correspondence between the vertices such that :

- 1) Corresponding sides are equal in length.
- 2) Corresponding angles are equal in measure.

#### **Example(1):**

The polygon BRAKE is congruent to the polygon CHOKE, the vertices are written in the same order.

Complete:

$$CH = \dots, EK = \dots$$

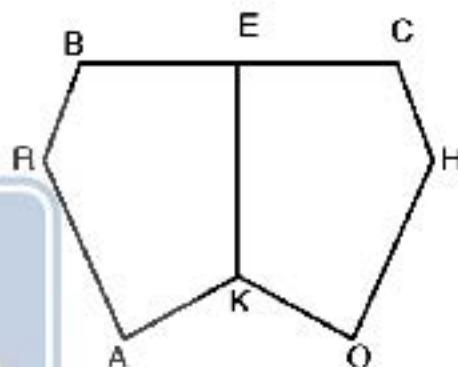
$$HO = \dots, EC = \dots$$

$$KO = \dots, KE \text{ is a common side}$$

$$m(\angle C) = m(\angle \dots), m(\angle OKE) = m(\angle \dots)$$

$$m(\angle H) = m(\angle \dots), m(\angle KEC) = m(\angle \dots)$$

$$m(\angle O) = m(\angle \dots)$$

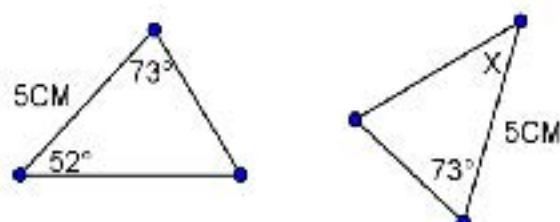


#### **Example(2):**

In the opposite figure :

These triangles are congruent ,

$$\text{then } X = \dots^\circ$$



Example(3):

a) The diagonal of the rectangle divides its surface into two ..... triangles.

b) If  $\triangle ABC \cong \triangle XYZ$ , then  $AB = \dots$  and  $m(\angle Z) = m(\dots)$

c) If c is the midpoint of  $\overline{AB}$ , Then  $\overline{AC} \dots \overline{BC}$

d) The two squares are congruent if ..... are equal .

e) The two rectangles are congruent if ..... are equal.



**Try by yourself**



- [1] The polygon BRAKE is congruent to the polygon CHOKE,  
the vertices are written in the same order.

\***Complete:**

$$CH = \dots, EK = \dots$$

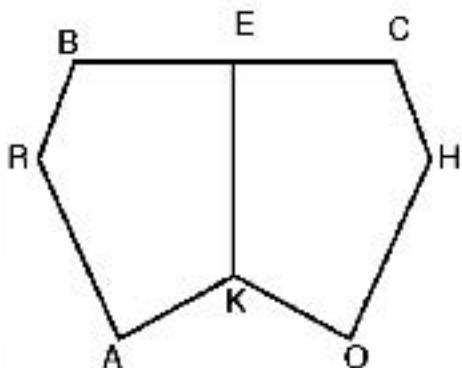
$$HO = \dots, EC = \dots$$

$$KO = \dots, KE \text{ is a common side}$$

$$m(\angle C) = m(\angle \dots), m(\angle OKE) = m(\angle \dots)$$

$$m(\angle H) = m(\angle \dots), m(\angle KEC) = m(\angle \dots)$$

$$m(\angle O) = m(\angle \dots)$$



- [2] The two pentagons shown are congruent.

**Complete**

[a] B Corresponds to ....

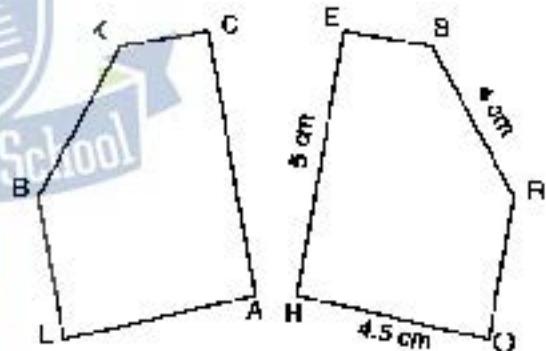
[b] The polygon BLACK is congruent to  
the polygon.....

[c] KB = ..... cm.

[d]  $m(\angle E) = m(\angle \dots)$

[e] CA = .....

[f]  $m(\angle A) = m(\angle \dots)$



- [3] Complete:

a) The diagonal of the rectangle divides its surface into two ..... triangles.

b) If  $\triangle ABC \cong \triangle XYZ$ , then  $AB = \dots$  and

$$m(\angle Z) = m(\dots)$$

### Congruent Triangles

#### The first case of congruence of two triangles (side – angle – side) (S.A.S)

Two triangles are congruent if two sides and the included angle of one triangle are congruent to the corresponding parts of the other triangle.

Example 1 :

If  $AB = AD = 7\text{ cm}$ ,  $BC = 7\text{ cm}$ ,

$$m(\angle BAC) = m(\angle DAC) = 25^\circ$$

$$\text{And } m(\angle B) = 30^\circ$$

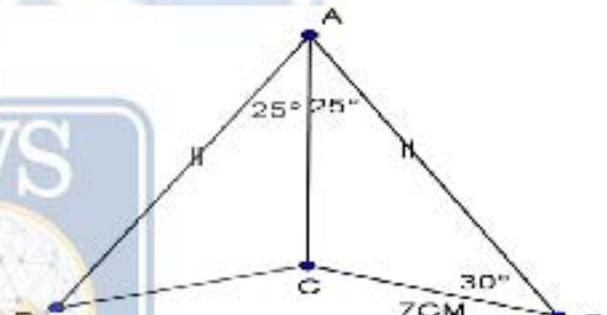
Complete the following :

1)  $\triangle ACB \equiv \triangle \dots$

2)  $m(\angle D) = \dots$

3)  $CD = \dots \text{ cm}$

4)  $m(\angle ACD) = \dots$

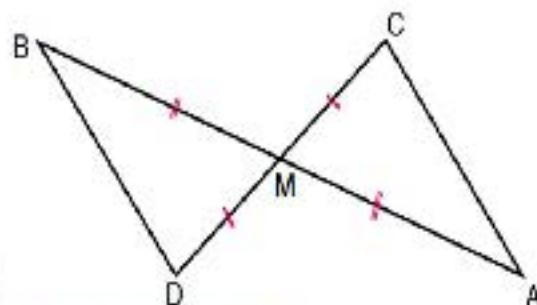


Example 2:

In the figure opposite,

$AB \cap CD = \{M\}$ ,  $AM = BM$ , and  $CM = DM$ .

Does  $\triangle AMC \cong \triangle BMD$ ? why?

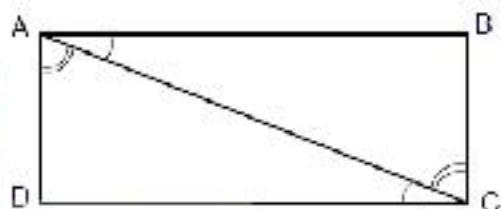


**The second case of congruence of two triangles (angle – side – angle)  
(A.S.A)**

Two triangles are congruent if two angles and the side drawn between their vertices of one triangle are congruent to the corresponding parts of the other triangle

[1] In the opposite figure:

Find that  $\triangle ABC \cong \triangle CDA$



[2] In the opposite figure:

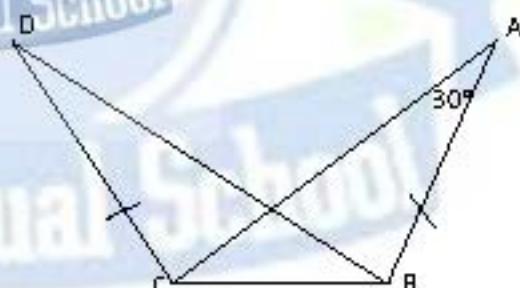
If  $AB = DC$ ,  $AC = DB$  and  $m(\angle A) = 30^\circ$ ,

Complete:

1)  $\triangle ABC \cong \triangle \dots$

2)  $m(\angle D) = \dots^\circ$

3)  $m(\angle DBC) = m(\angle \dots)$



Important note

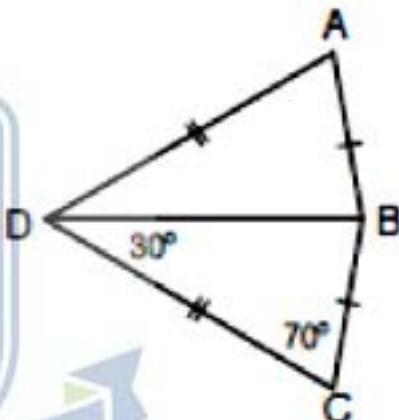
\*The diagonal of the rectangle divides its surface into two **congruent triangles**.

[1] mention two cases of congruency of two triangles.

[2] In the figure opposite :

$AB = BC, AD = CD, m(\angle C) = 70^\circ,$

$m(\angle BDC) = 30^\circ.$  find  $m(\angle ABD).$



### The third case of congruence of two triangles (side – side – side) (S.S.S)

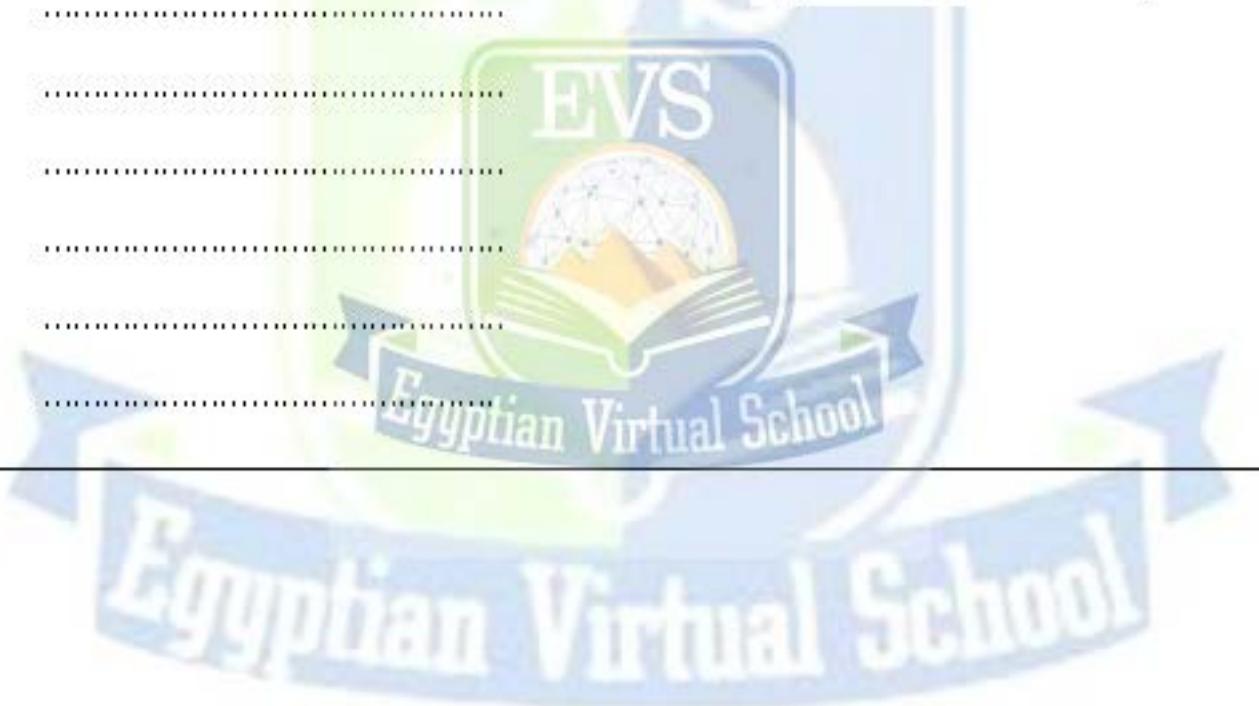
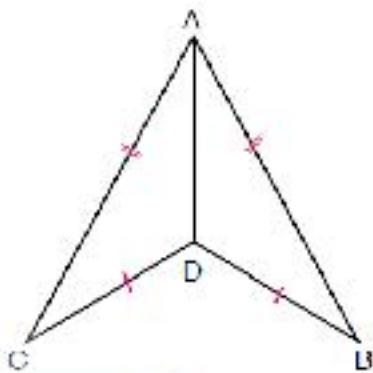
Two triangles are congruent if each side of one triangle is congruent to the corresponding side of the other triangle

[1] In the figure opposite,

$$AB = AC, BD = CD$$

$$\text{Is } \triangle ADB \cong \triangle ADC$$

verify that:  $\overrightarrow{AD}$  bisects  $\angle A$

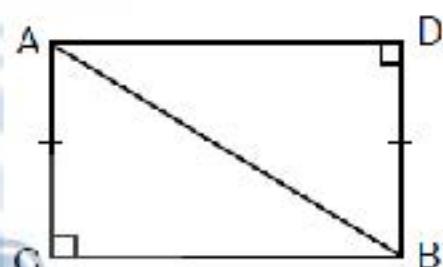


The fourth case of congruence of two triangles(Hypotenuse and one side in the right – angled triangle R.H.S)

Two right – angled triangles are congruent if the hypotenuse and a side of one triangle are congruent to the corresponding parts of the other triangle

[1] In the figure opposite:

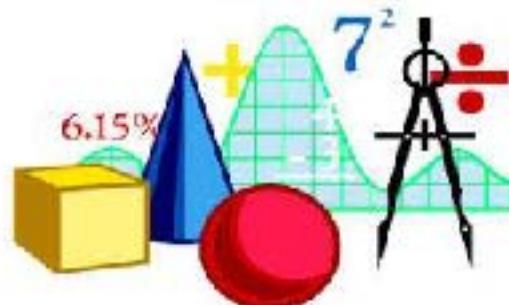
Find the two congruent triangles.



**\* Mark (✓) for the correct statement:**



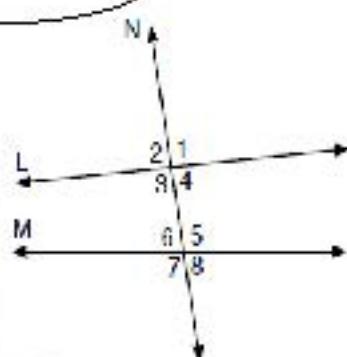
- [a] Two triangles are congruent if the lengths of sides of one triangle are equal to the corresponding parts of the other.
- [b] Two triangles are congruent if the measures of the angles of one triangle are equal to the measures of the corresponding parts of the other.
- [c] Two right-angled triangles are congruent if the lengths of two sides of one triangle are equal to the corresponding parts of the other triangle.
- [d] Two right-angled triangles are congruent if the length of the hypotenuse and the measure of an angle differs from the right angle are equal to the corresponding parts of the other triangle.
- [e] Two right-angled triangles are congruent if the length of the hypotenuse and the length of a side of one triangle are equal to the corresponding parts of the other triangle.



## Parallelism

\* If a straight line intersects two parallel straight lines, then:

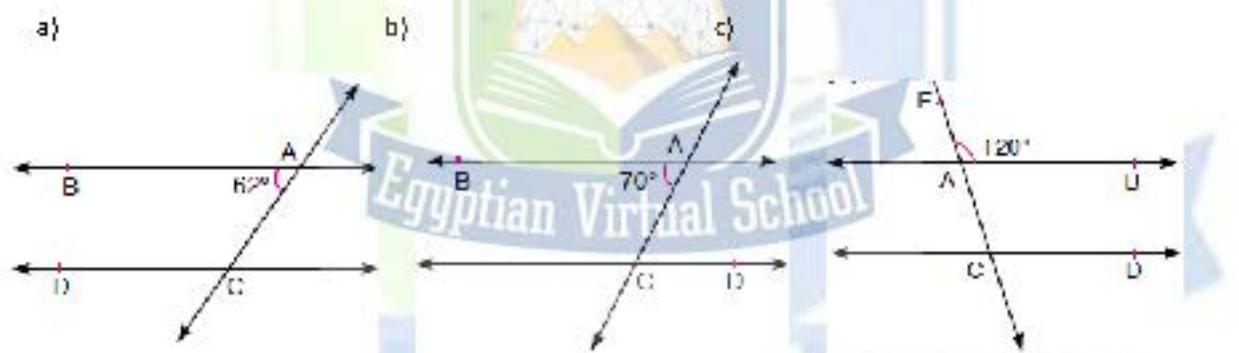
1) Every two alternate angles are equal in measure.



2) Every two corresponding angles are equal in measure.

3) Every two interior angles on one side of the transversal are supplementary.

[1] Complete:



$$m(\angle ACD) = \dots^\circ \quad \dots^\circ$$

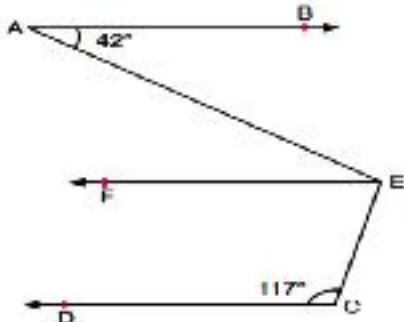
$$m(\angle ACD) = m(\angle \dots) \quad \dots^\circ$$

$$m(\angle ACD) = m(\angle \dots) \quad \dots^\circ$$

[2] In the figure opposite,

$$\overrightarrow{AB} \parallel \overrightarrow{CD}, \overrightarrow{EF} \parallel \overrightarrow{CD},$$
$$m(\angle A) = 42^\circ, \text{ and } m(\angle C) = 117^\circ$$

Determine  $m(\angle AEC)$

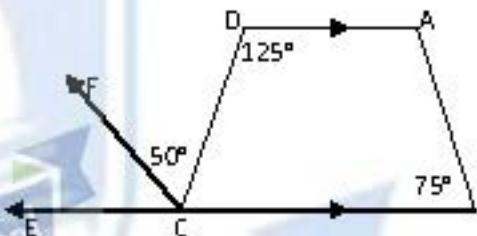


[3] In the opposite figure:

$$\overrightarrow{AD} \parallel \overrightarrow{BC}, E \in BC,$$

$$m(\angle B) = 75^\circ, m(\angle D) = 125^\circ \text{ and } m(\angle DCF) = 50^\circ$$

Is  $\overrightarrow{AB} \parallel \overrightarrow{CF}$ ? why?



Try by yourself:



1) If two straight lines are parallel to a third straight line , then they are .....

2) If a straight line cuts two parallel straight lines , then each two corresponding angles are .....

3) If a straight line cuts two parallel straight lines , then each two interior angles in the same side of the transversal are .....

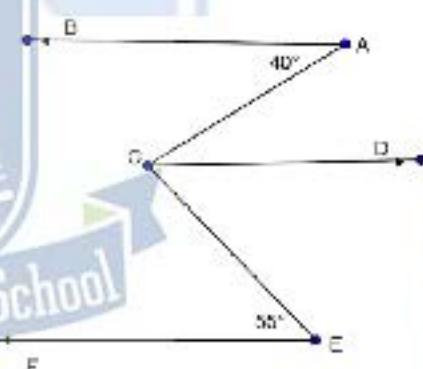
4) In the opposite figure :

$$m(\angle A) = 40^\circ, m(\angle E) = 55^\circ$$

$\overrightarrow{AB} \parallel \overrightarrow{EF}$  and  $\overrightarrow{AB} \parallel \overrightarrow{CD}$

Find :

$$m(\angle ACE)$$

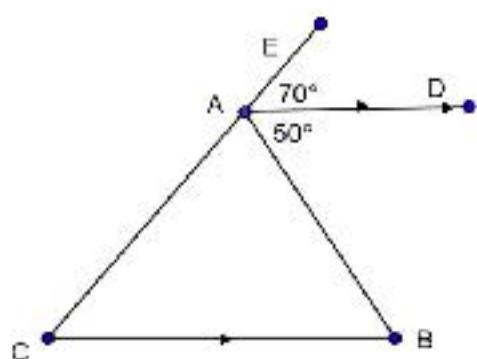


5- In the opposite figure :

$$AD \parallel BC, E \in CA,$$

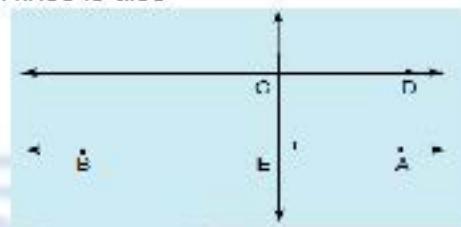
$$m(\angle DAE) = 70^\circ \text{ and } m(\angle DAB) = 50^\circ$$

Find the measures of the angles of the triangle ABC

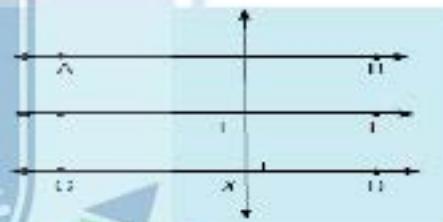


### **Important**

- 1) A straight line that is perpendicular to one of two parallel lines is also perpendicular to the other.

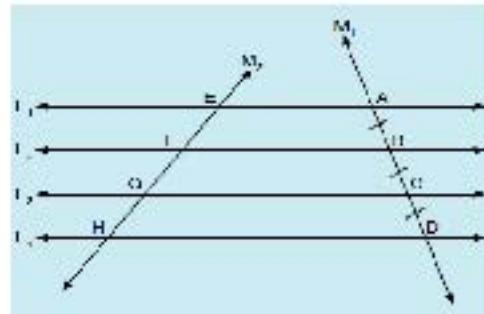


- If each one of two straight lines is perpendicular to a third line in a plane, then the two straight lines are parallel.



- 3) If two straight lines are parallel to a third straight line, then these two straight lines are parallel to each other.

- 4) If Parallel straight lines divide a straight line into segments of equal lengths, then they divide any other straight line into segments of equal lengths



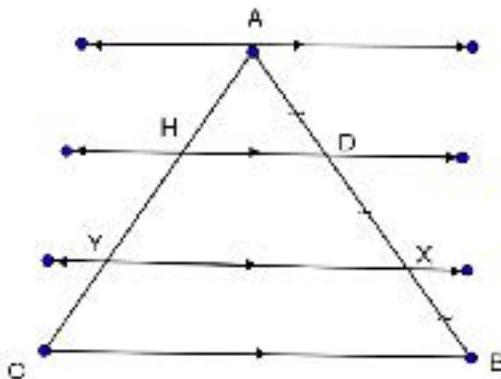
**[1] In the opposite figure :**

$$\overline{AO} \parallel \overline{HD} \parallel \overline{YX} \parallel \overline{CB}$$

$$AD = DX = XB$$

and  $AC = 18 \text{ cm}$ .

Find the length of  $\overline{AY}$



**[2] In the opposite figure :**

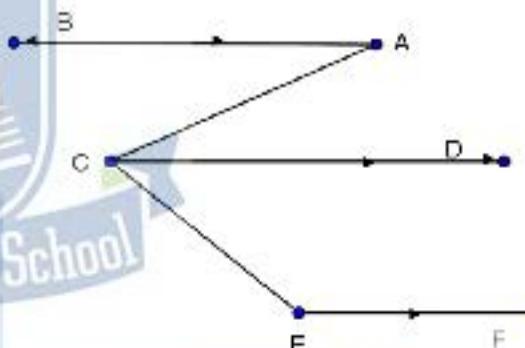
$$\overline{AB} \parallel \overline{CD} \parallel \overline{EF}, m(\angle A) = 35^\circ \text{ and}$$

$\overline{CD}$  bisects  $\angle ACE$

**Find :**

$$1) m(\angle DCE)$$

$$2) m(\angle CEF)$$

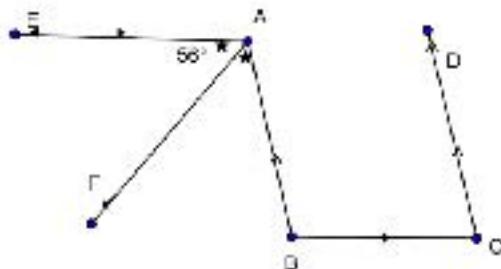


**[3] In the opposite figure :**

$$\overline{AE} \parallel \overline{CB}, \overline{BA} \parallel \overline{CD}$$

$\overrightarrow{AF}$  bisects  $\angle BAE$  and  $m(\angle EAF) = 56^\circ$

Find :  $m(\angle C)$



**[4] In the opposite figure :**

$$\overline{XL} \parallel \overline{YZ}, \overline{XY} \parallel \overline{LZ}$$
 and

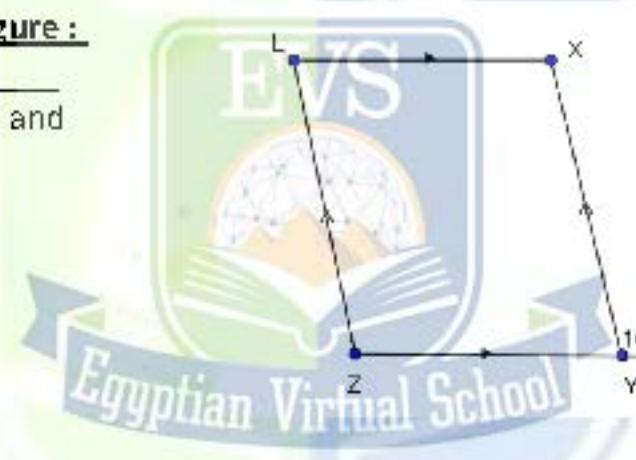
Where  $M \in \overrightarrow{ZY}$

**Find :**

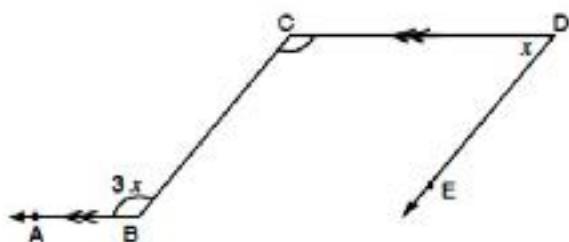
1)  $m(\angle X)$

2)  $m(\angle Z)$

3)  $m(\angle L)$

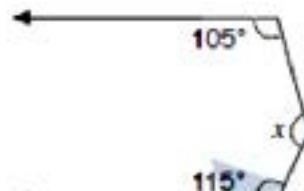


[5] In the opposite figure, if  $\overline{CD} \parallel \overline{BA}$  and  $\overline{DE} \parallel \overline{CB}$ , then  $x = \dots \dots$

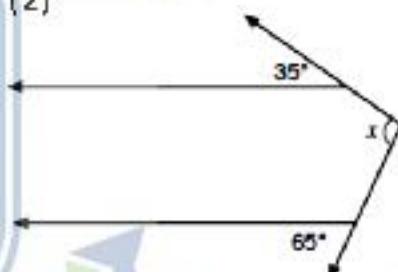


[6] Find the measure of ( $\angle X$ ) In each of the following:

(1)



(2)



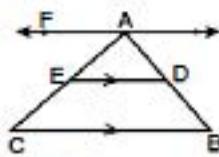
**[7] Choose the correct answer:**

(1) If two straight lines are on the same plane and do not intersect, then they are .....

- (a) skew      (b) perpendicular      (c) parallel      (d) congruent

(2) In the opposite figure :

$$AD : DB = \dots$$



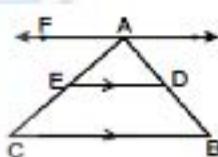
- (a) 1 : 1      (b) 1 : 2

- (c) 1 : 3

- (d) 1 : 4

(3) In the opposite figure :

$$AD : AB = \dots$$



- (a) 1 : 1

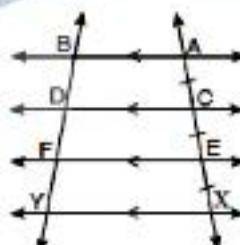
- (b) 1 : 2

- (c) 1 : 3

- (d) 1 : 4

(4) In the figure opposite :

If  $BF = 2 \text{ cm}$  then  $BY = \dots \text{ cm}$



## **Geometric Constructions**

1) Using the geometric instruments , draw an angle of measure  $120^\circ$

and bisect it

( Don't remove the arcs )



2) Using the geometric instruments , draw an angle of measure  $120^\circ$

and divide it into four congruent angles

( Don't remove the arcs )



3) Draw an angle whose vertex is A and its measure is  $130^\circ$  , use a ruler and a compasses to divide the angle A into 4 equal angles in measure  
( Don't remove the arcs ).

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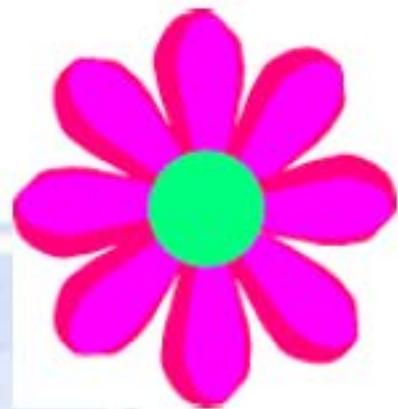


4) Using the ruler and the compasses , draw  $\triangle ABC$  in which  
 $AB = AC = 5 \text{ cm}$  ,  $BC = 6 \text{ cm}$  , then draw  $AD \perp BC$  where  
 $AD \cap BC = \{D\}$  . Then find the length of  $AD$   
( Don't remove the arcs )

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5) Using the ruler and the compasses , draw  $\triangle ABC$  in which  
 $AB = AC = 3 \text{ cm}$ ,  $BC = 5 \text{ cm}$  then bisect  $\angle A$  by the bisector  
 $\overline{AD}$  where  $D \in \overline{BC}$  ( Don't remove the arcs ).

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5) Using the ruler and the compasses , draw  $\triangle XYZ$  in which:  
 $m(\angle X)=50^\circ$  , $m(\angle Y)=70^\circ$  ,then draw  $\overline{ZL} \perp \overline{XY}$  to cut it at L.  
then find:

- 1) The length of  $\overline{ZL}$
- 2)The area of  $\triangle XYZ$

.....  
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- 6) Using the ruler and the compasses, draw the equilateral triangle ABC of side length 6cm. Bisect each of A, B and C to intersect at M  
prove that  $MA = MB = MC$

(Don't remove the arcs)

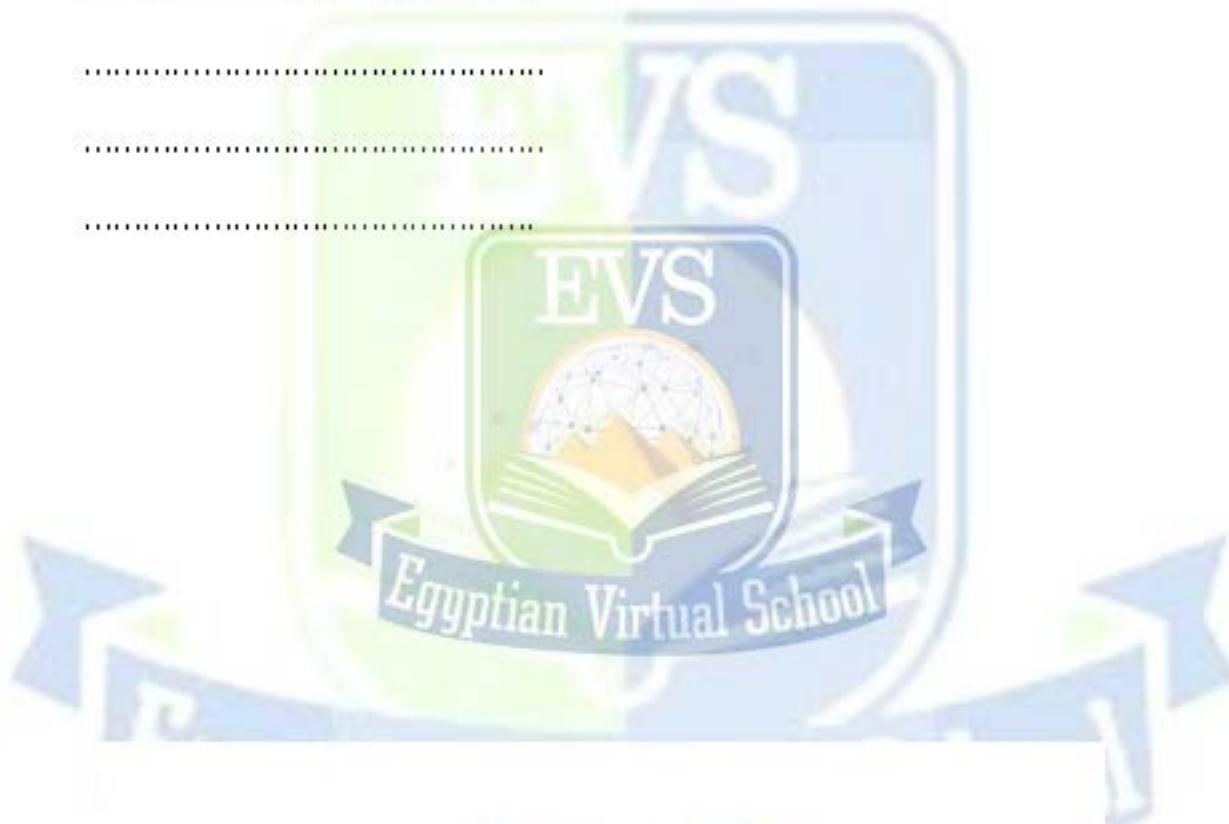
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- 7) Using the geometric instruments, Draw AB where  $AB = 6\text{ cm}$ , Draw the axis of symmetry of AB, take C  $\in$  the axis of symmetry and of distance 4 cm from AB.What is the type of  $\triangle ABC$  according to its sides?

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8) Using the ruler and the compasses, Draw  $\triangle ABC$  in which  
 $AB = AC = 5 \text{ cm}$  and  $BC = 6 \text{ cm}$ , then draw  $\overline{AD} \perp \overline{BC}$  where  
 $\overline{AD} \cap \overline{BC} = \{D\}$ , then find by measuring the length of  $\overline{AD}$ .  
(Don't remove the arcs)

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## Geometric concepts

### [1] Complete each of the following:

- 3) The angle is .....
- 4) The measure of the straight angle is .....° and the measure of zero angle is .....°
- 5) The measure of the right angle = .....°
- 6) The measure of the acute angle is less than .....° and more than .....°
- 7) The measure of the obtuse angle is less than .....° and more than .....°
- 8) The angle whose measure is greater than  $180^{\circ}$  but less than  $360^{\circ}$  is called .....
- 9) The angle whose measure is  $179^{\circ}$ , its type is .....
- 10) The two complementary angles are two angles whose sum of their measures is .....°
- 11) The complements of the equal angles in measure are .....
- 12) The two supplementary angles are two angles whose sum of their measures is .....°
- 13) The two adjacent angles formed by a straight line and a ray with a starting point on this straight line are .....
- 14) If the two outer sides of two adjacent angles are perpendicular then these two angles are .....
- 15) The two adjacent angles are complementary ,then their outer sides are .....
- 16) If the two outer sides of two adjacent angles are on the same straight line then these two angles are .....
- 17) The two adjacent angles are supplementary ,then their outer sides are .....
- 18) The angle of measure  $43^{\circ}$  complements angle of measure .....° and supplements angle of measure .....°
- 19) The angle of measure ..... complements an angle of measure  $50^{\circ}$  and supplements angle of measure  $140^{\circ}$
- 20) The acute angle complements .....angle and supplements .....angle.
- 21) Zero angle is complemented by .....angle and supplemented by .....angle.
- 22) The obtuse angle supplements .....angle.
- 23) If two straight lines intersect, then each two vertically opposite angles are .....

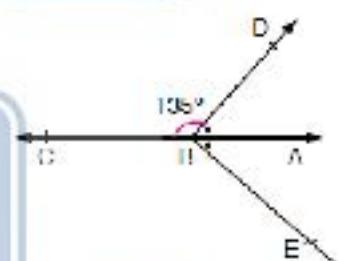
- 24) The right angle is vertically opposite angle to .....
- 25) The measure of an angle which equivalent to two right angles = .....° and it's called .....
- 26) If  $m(\angle B) = 160^\circ$ , then  $m(\text{reflex } \angle B) = \dots \dots \dots$  °
- 27) If  $\angle A$  supplements  $\angle B$  and  $\angle A \equiv \angle B$ , then  $m(\angle B) = \dots \dots \dots$  °
- 28) the angle whose measure is  $46^\circ$  vertically opposite to an angle whose measure is .....
- 29) The sum of the measures of the accumulative angles at a point equals .....

**[2] In the figure opposite,**

If  $B \in AC$ ,  $m(\angle DBC) = 135^\circ$  and

$BA$  bisects  $\angle DBE$  find:

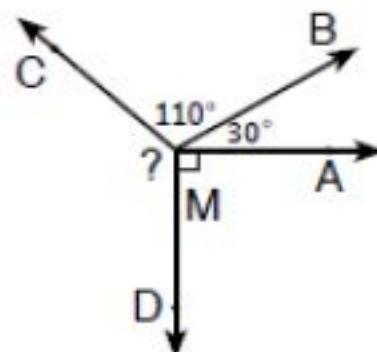
$m(\angle ABD)$ ,  $m(\angle DBE)$ ,  $m(\angle CBE)$



**[3] In the figure opposite :**

$m(\angle AMB) = 30^\circ$ ,  $m(\angle BMC) = 110^\circ$  and

$m(\angle AMD) = 90^\circ$ . Find  $m(\angle CMD)$ .



## The congruency & the congruent triangles

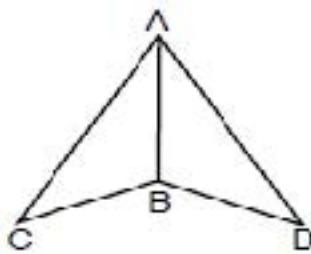
### 1] Complete each of the following:

- 1) Two angles are congruent if .....
- 2) Two line segments are congruent if .....
- 3) Two polygons are congruent if .....
- 4) Two polygons are congruent if there is a correspondence between their vertices such that .....
- 5) mention two cases of congruency of two triangles.
- 6) The two adjacent angles formed by a straight line and a ray with a starting point on this straight line are .....
- 7) If  $\overline{AB} \equiv \overline{CD}$ , then  $AB - CD = \dots$
- 8) If Z is the midpoint of  $\overline{XY}$ , then  $\overline{XZ} \dots \overline{YZ}$
- 9) If the ratio between two supplementary angles is 4 : 5, then the measure of the greater angle is .....
- 10) If  $m(\angle A) = 2 m(\angle B)$ ,  $\angle A$  complements  $\angle B$ , then  $m(\angle A) = \dots$
- 11) The two triangles are congruent if .....
- 12) The two triangles are congruent if two sides and ..... are congruent with their corresponding in the other triangle.
- 13) The two triangles are congruent if two angles and ..... are congruent with their corresponding in the other triangle.
- 14) Two right angled triangles are congruent if .....
- 15) The diagonal of the rectangle divides its surface into two ..... triangles.
- 16) If the two triangles ABC and DEF are congruent, then  $:BC = \dots$ ,  $m(\angle E) = m(\angle \dots)$
- 17) If  $DE = XY$ ,  $DF = XZ$  and  $m(\angle D) = m(\angle X)$ , then  $\triangle \dots, \dots$  are congruent.
- 18) The two triangles XYZ and MNL are congruent, if  $YZ = 8 \text{ cm}$ ,  $m(\angle Y) = 40^\circ$  then in the other triangle : ..... = 8 cm,  $m(\angle \dots) = 40^\circ$
- 19) If  $AB = DF = 5 \text{ cm}$ ,  $AC = DE = 7 \text{ cm}$ ,  $m(\angle A) = m(\angle D) = 55^\circ$  then the two triangles ABC, DFE are congruent with .....
- 20) In  $\triangle ABC$  : if  $m(\angle B) = 3 m(\angle A) = 90^\circ$ , then  $m(\angle C) = \dots$
- 21)  $\triangle ABC$  is congruent to  $\triangle XYZ$ ,  $m(\angle A) + m(\angle B) = 140^\circ$ , then  $m(\angle Z) = \dots^\circ$



[2] In the figure opposite :

If  $\Delta ABC \cong \Delta ABD$  let the perimeter of the figure  
 $ACBD = 20 \text{ cm}$  and  $AB = 6 \text{ cm}$ ,  
then perimeter of  $\Delta ABC = \dots \text{ cm}$ .



[3] In the figure opposite :

$$m(\angle BAD) = m(\angle BCD) = 90^\circ$$

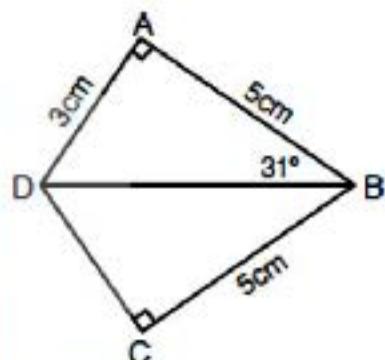
$$m(\angle ABD) = 31^\circ,$$

$$AB = CB = 5 \text{ cm}, AD = 3 \text{ cm}$$

(a) Prove that :

the two triangles ABD and CBD are congruent.

(b) find the length of CD.



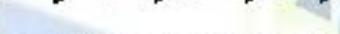
## The parallelism



### 1] Complete each of the following:

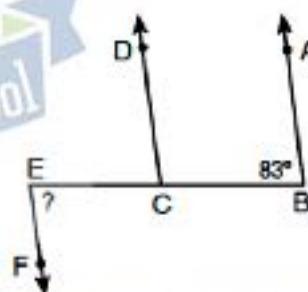
- 1) When a transversal cuts two parallel lines, Then :  
The alternate angles are .....  
The corresponding angles are .....  
The interior angles on the same side of the transversal are .....
- 2) A straight line that is perpendicular to one of two parallel lines is also ..... to the Other.
- 3) A straight line that is parallel to one two parallel lines is also ..... to the other.
- 4) A straight line that is perpendicular to one of two parallel lines is .....
- 5) The two straight lines perpendicular to a third one are .....

### 2] In the figure opposite :



$BA \parallel CD$ ,  $CD \parallel EF$  and

$m(\angle ABC) = 83^\circ$  find  $m(\angle CEF)$ .



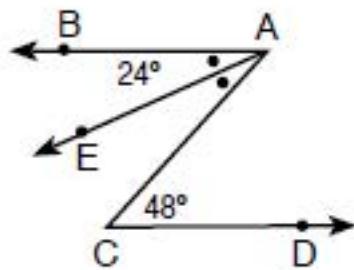
[3] In the figure opposite :

AE bisects  $\angle BAC$ ,  $m(\angle BAE) = 24^\circ$

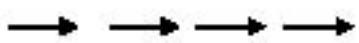
$m(\angle ACD) = 48^\circ$ . Complete :

First :  $m(\angle BAC) = \dots \text{ } ^\circ$

Second :  $\overrightarrow{AB} // \dots \rightarrow$



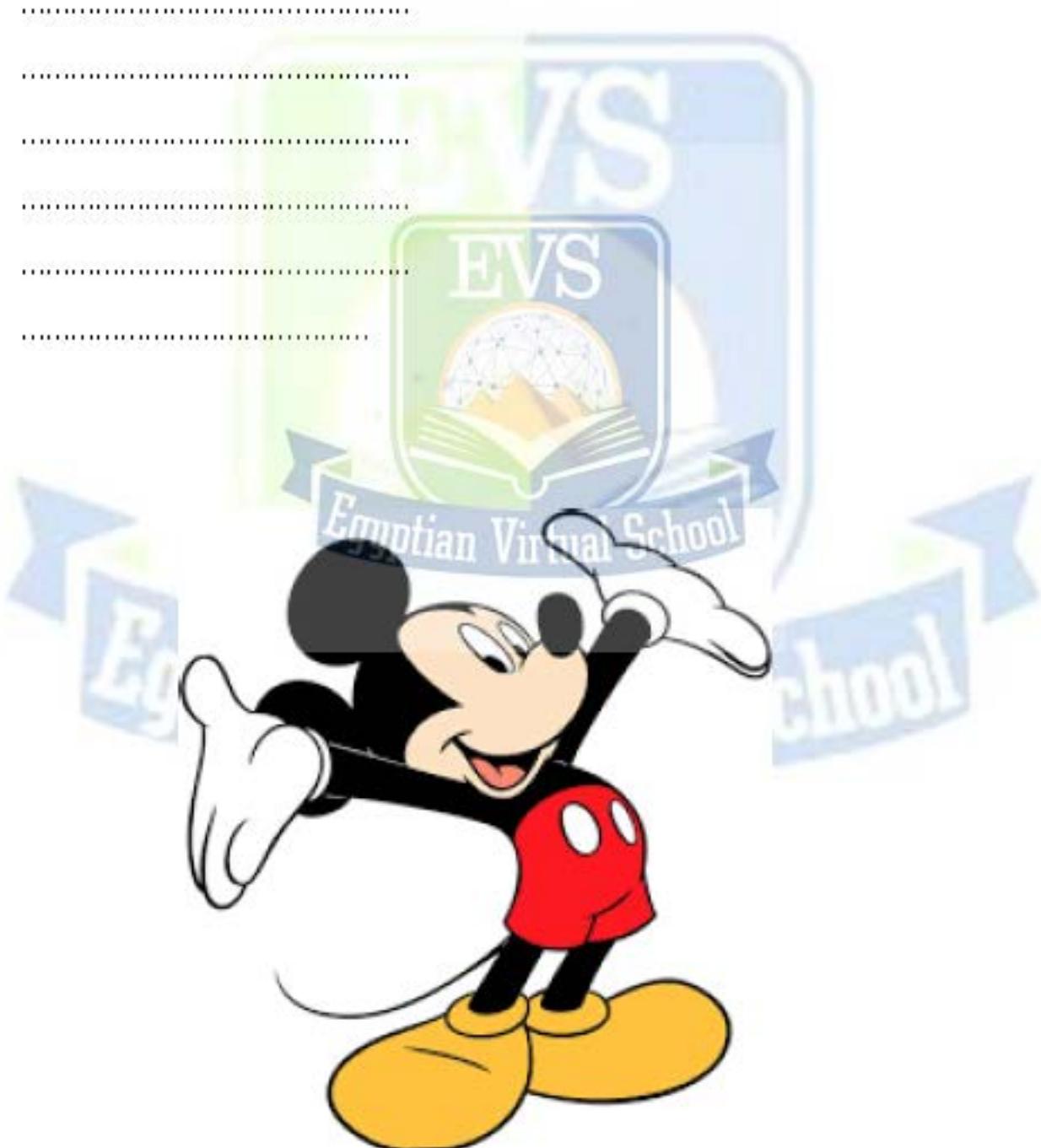
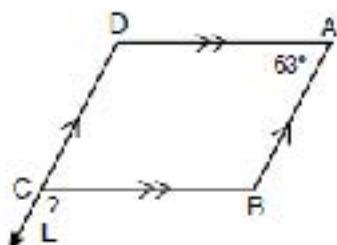
[4] In the figure opposite :



$AB \parallel DC$ ,  $AD \parallel BC$  and

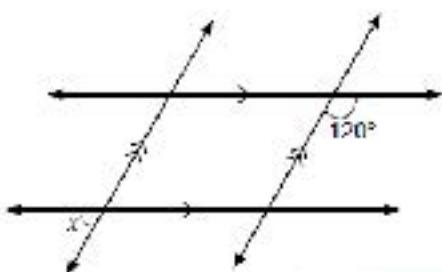
$m(\angle BAD) = 63^\circ$

find  $m(\angle BCE)$ .

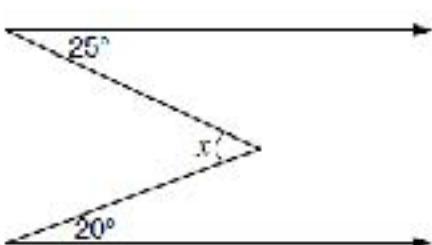


**[5] Find the value of x in each figure:**

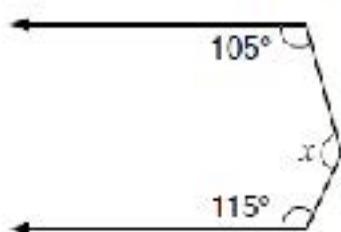
a)



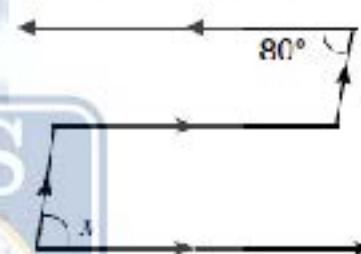
b)



c)



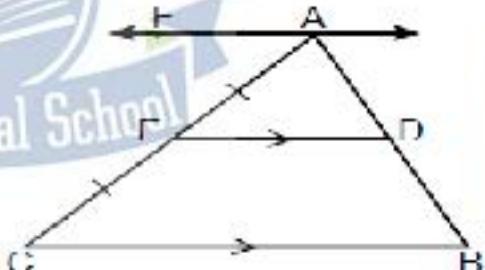
d)



[6] In the figure opposite :

If  $AB = 3 \text{ cm}$

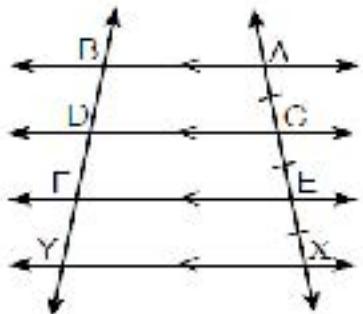
then BD = ..... cm



[7] In the figure opposite :

If  $BF = 2 \text{ cm}$

then BY = ..... cm

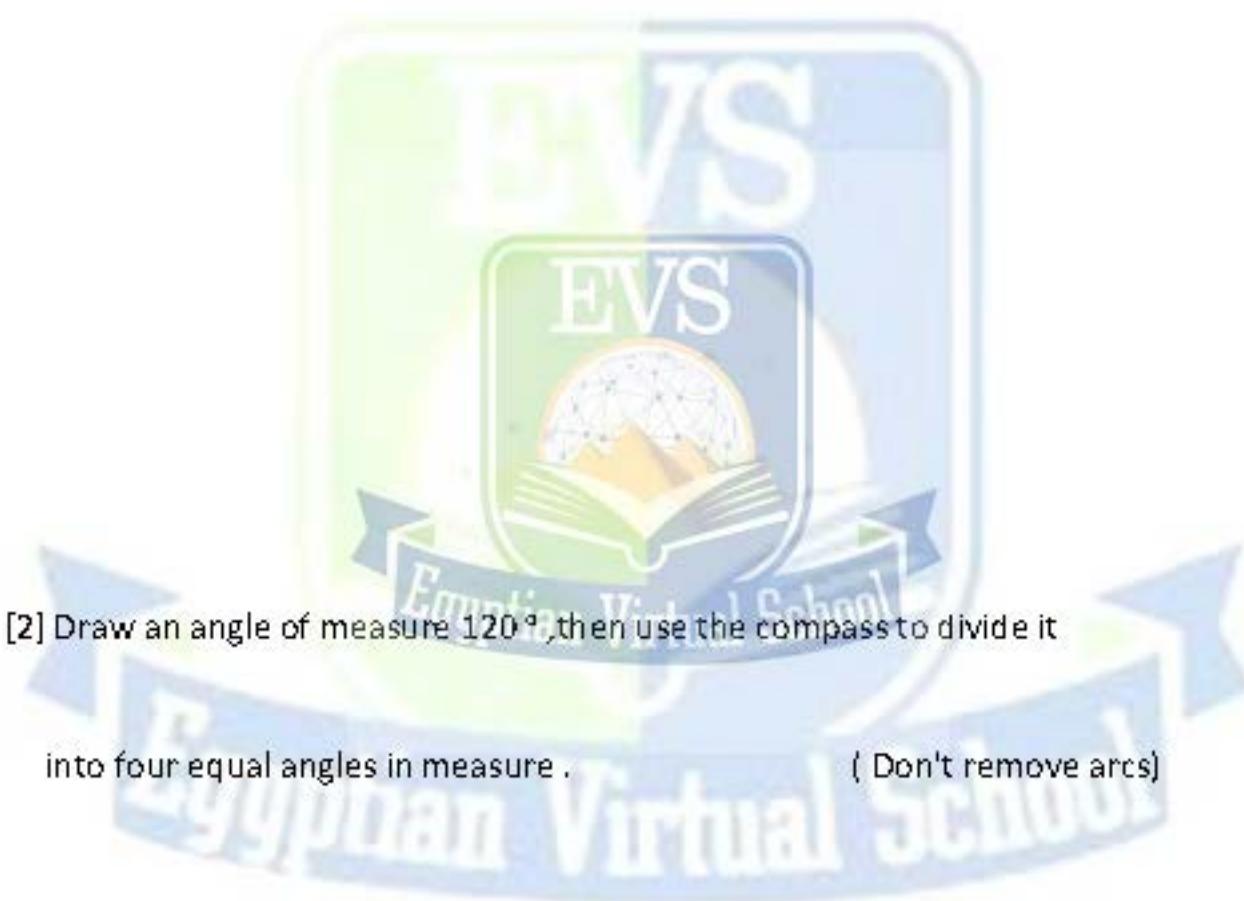


## Geometric constructions

### \*Constructing the bisector of a given angle

[1] Draw an angle of measure  $80^\circ$ , then use the compass to bisect it.

( Don't remove arcs)



[2] Draw an angle of measure  $120^\circ$ , then use the compass to divide it

into four equal angles in measure .

( Don't remove arcs)

[3] Using the ruler and the compass ,draw the equilateral triangle ABC of side length 6cm. Bisect each of  $\angle A$ ,  $\angle B$ ,  $\angle C$  by bisectors intersecting at M . What is the relation between MA, MB and MC?



# **Summary of geometry**

## **for prep one (1<sup>st</sup> term)**

- 1) The sum of measures of two complementary angles equals 90**
- 2) The sum of measures of two supplementary angles equals 180**
- 3) The measure of the straight angle is 180**
- 4) The measure of the right angle is 90**
- 5) The two adjacent angles formed by a straight line and a ray with a starting point on this straight line are supplementary**
- 6) The outer sides of the two supplementary adjacent angles are on the same straight line**
- 7) If the two adjacent angles are not supplementary , then their outer sides are not on the same straight line**
- 8) If the two adjacent angles are complementary , then their outer sides are perpendicular**
- 9) The sum of measures of the accumulative angles at a point = 360**
- 10) The two adjacent angles in which the two outer sides are on the same straight line are supplementary**
- 11) If two straight line intersect, then each two vertically opposite angles are equal in measure**
- 12) In the right angled triangle, the area of the square drawn on its hypotenuse is equal to the sum of the areas of the squares drawn on the other two sides**
- 13) The acute angle is supplemented by an obtuse angle**
- 14) The right angle is supplemented by a right angle**
- 15) If two angles are supplementary , then one of them is an acute and the other is an obtuse or both of them are right angles .**

**16) Two triangles are congruent :**

- 1- if two sides and the included angle of the first triangle are congruent to their corresponding from the other triangle
- 2- if two angles and the included side of the first triangle are congruent to their corresponding from the other triangle
- 3- If each side of the first triangle is congruent to its corresponding from the other triangle
- 4- The two right angled triangle are congruent if the hypotenuse and a side of one triangle are congruent to their corresponding from the other triangle

**17) If a straight line intersects two parallel straight lines then each two alternate angles are equal in measure**

**18) If a straight line intersects two parallel straight lines then each two corresponding angles angles are equal in measure**

**19) If a straight line intersects two parallel straight lines then each two interior angles at one side of the transversal are supplementary**

**20) If a straight line intersect two straight lines and two alternate angles are equal in measure then the two lines are parallel**

**21) If a straight line intersect two straight lines and two corresponding angles are equal in measure, then the two lines are parallel**

**22) If a straight line intersect two straight lines and two interior angles at one side of the transversal are supplementary , then the two lines are parallel**

**23) If two straight lines are parallel to a third straight line , then the two straight lines are parallel**

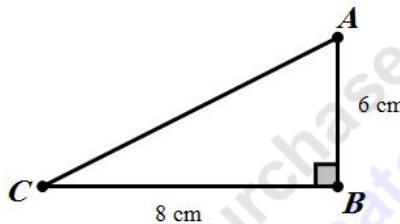
**24) The two perpendicular straight lines to a third line are parallel**

- 25) The perpendicular straight line to one of two parallel straight line is perpendicular to the other
- 26) If parallel straight lines divide a straight line into segments of equal lengths, then they divide any other straight line into segments of equal lengths.
- 27) The supplements of one angle are equal in measures
- 28) The complements of one angle are equal in measures
- 29) The acute angle complements an acute angle
- 30) If the triangle ABC is right-angled triangle at B, then

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$(AB)^2 = (AC)^2 - (BC)^2$$

$$(BC)^2 = (AC)^2 - (AB)^2$$



Best wishes

Mr Gamal Serag

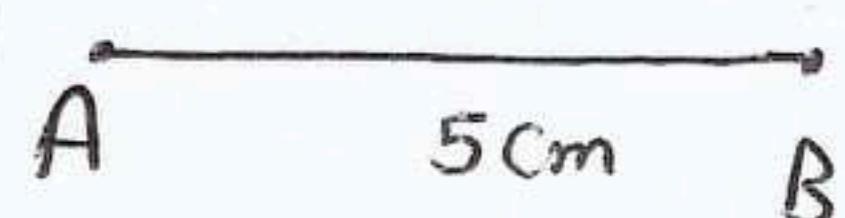
# Geometry

## Unit 4 : Geometry and measurement

- ① Geometric concepts and the relations between angles.
- ② Congruence.
- ③ Parallelism.
- ④ Geometric constructions.

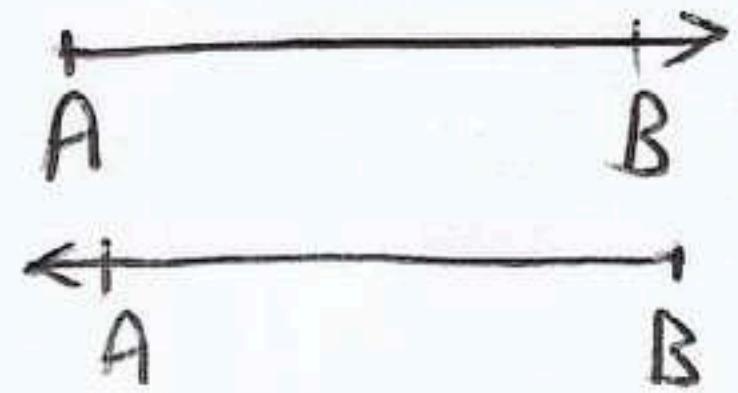
### I Geometric concepts and the relations between the angles.

\* **The line segment** is a set of points consisting of two distinct points and all points between them when we join them by a ruler, it has two end points  $\overline{AB}$  or  $\overline{BA}$  and its length is 5cm ( $AB=5\text{cm}$ )



\* **The ray** is a line segment extended from only one of its terminals without limit, it has a starting point and it hasn't end point

$$\overrightarrow{AB} \neq \overrightarrow{BA} \text{ and it has no length } \overline{AB} \subset \overrightarrow{AB}$$

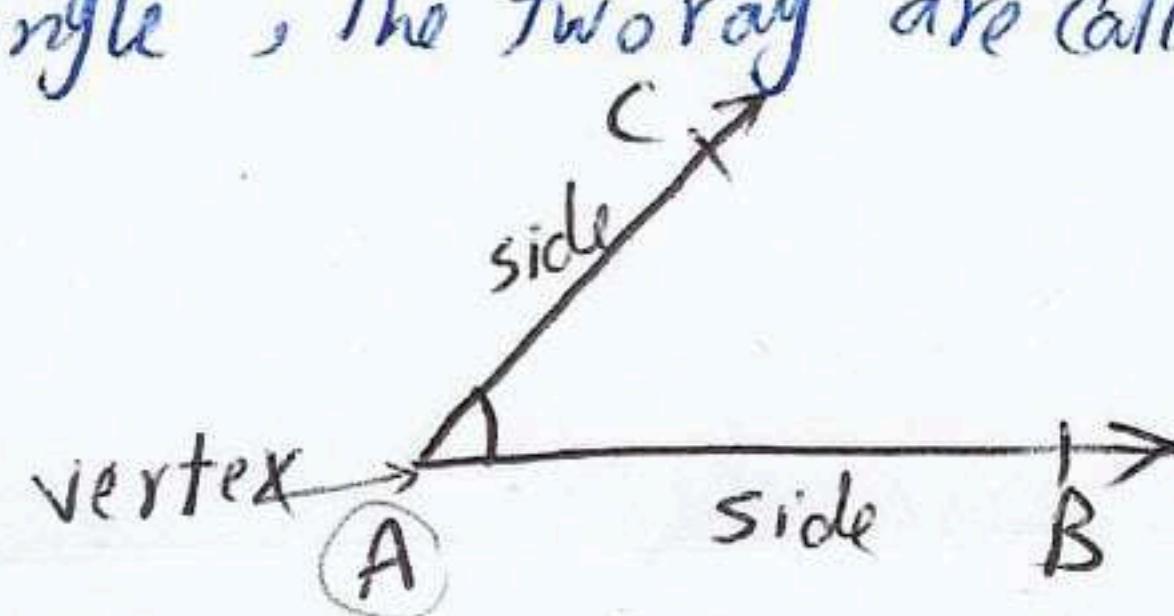


\* **The straight line** is a line segment extended from both directions infinitely, it hasn't a starting and hasn't ending point

$$\overleftrightarrow{AB} \text{ or } \overleftrightarrow{BA} \text{ it has no length. } \overline{AB} \subset \overleftrightarrow{AB} \subset \overleftrightarrow{BA}$$

\* **The angle** is the union of two rays with the same starting point and this point is called the vertex of the angle, the two rays are called two sides of the angle

$$\angle A \text{ or } \angle BAC \text{ or } \angle CAB$$



## The types of angles

## its measure

① Zero angle

its measure =  ${}^{\circ} 0$

its sides are coincident



② Acute angle

${}^{\circ} 0 < \text{its measure} < {}^{\circ} 90$

for example  $32^{\circ} 30'$  &  $60^{\circ}$

its measure =  ${}^{\circ} 90$

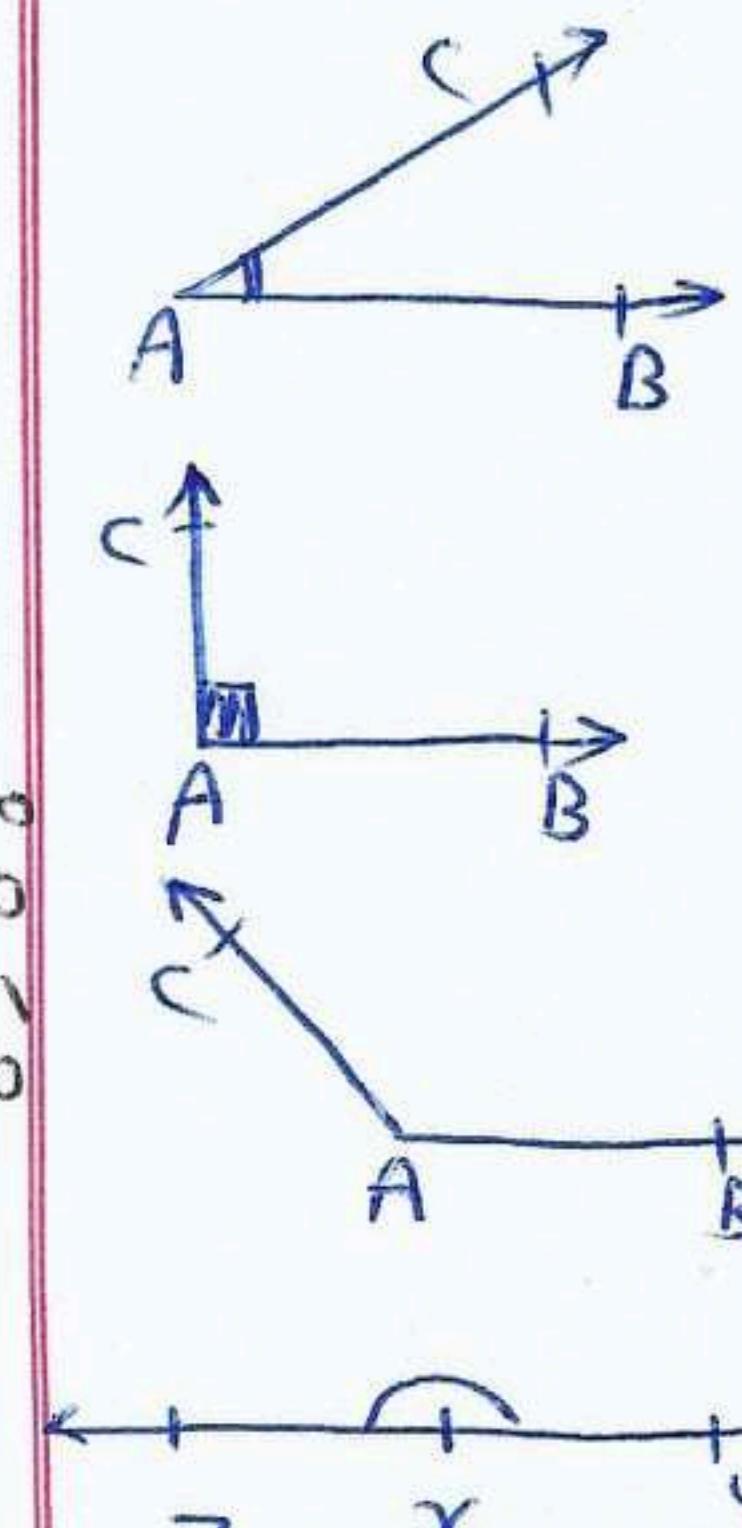
or  $89^{\circ} 60'$  ( $60' = 1^{\circ}$ )

${}^{\circ} 90 < \text{its measure} < {}^{\circ} 180$

for example  $100^{\circ}$  &  $179^{\circ} 30'$

its measure =  ${}^{\circ} 180$

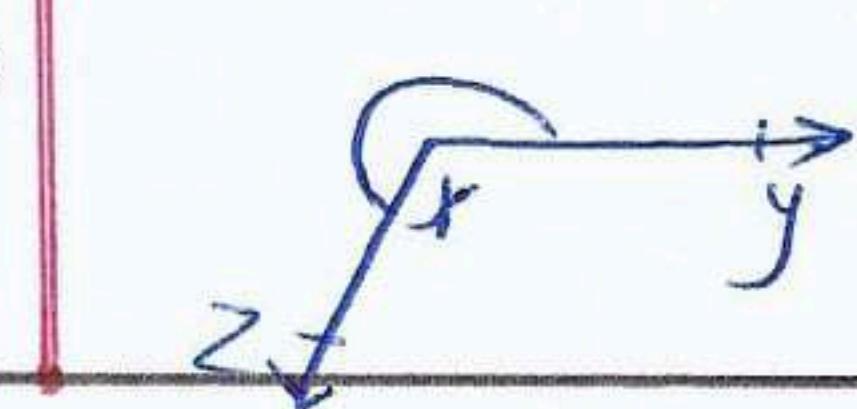
its sides are forming one straight line



⑤ straight angle

${}^{\circ} 180 < \text{its measure} < 360$

for example  $200^{\circ}$  &  $350^{\circ}$



$$m(\angle YXZ) + m(\text{reflex } \angle YXZ) = 360^{\circ}$$

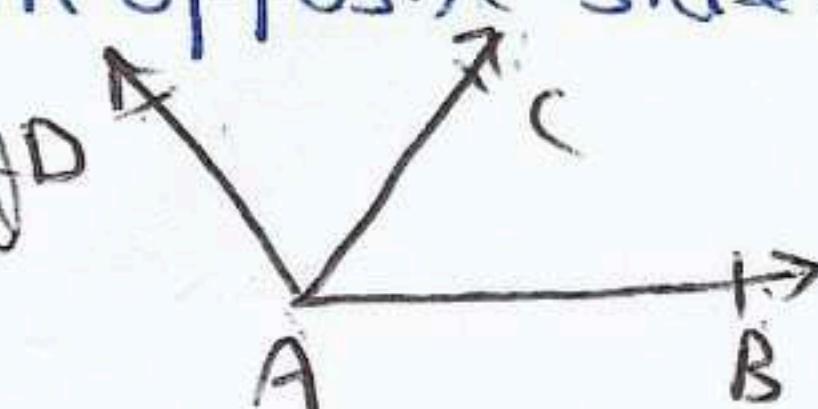
## The relations between the angles.

### ① Adjacent angles

Eng / Jana Ahmed

Two angles are said to be adjacent if they have a common vertex

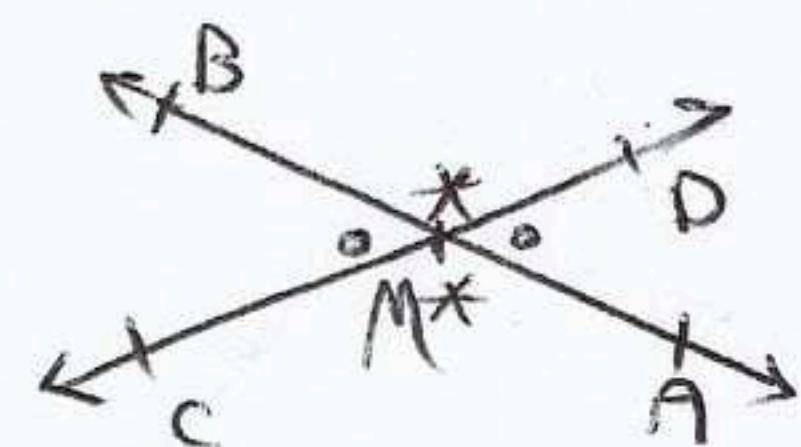
and a common side and the other two side are on opposite sides  
of this common side  $\angle CAB$  and  $\angle CAD$  adjacent



### ② Vertically opposite angles (V.O.A)

If two straight lines intersect, then the measures of each two vertically opposite angles are equal.

$$m(\angle DMA) = m(\angle BMC) \quad \& \quad m(\angle DMB) = m(\angle AMC)$$

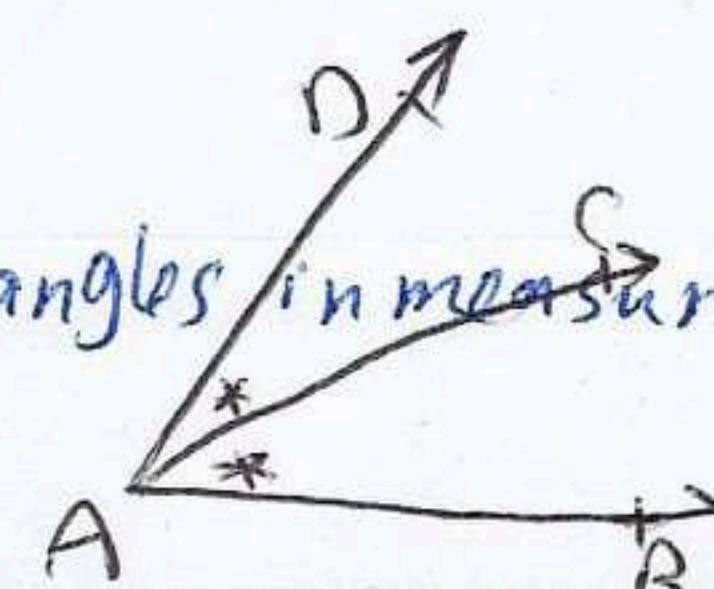


### ③ The angle bisector

it is the ray that divides the angle into two equal angles in measure

$$m(\angle CAB) = m(\angle CAD) = \frac{1}{2} m(\angle BAD)$$

[2]

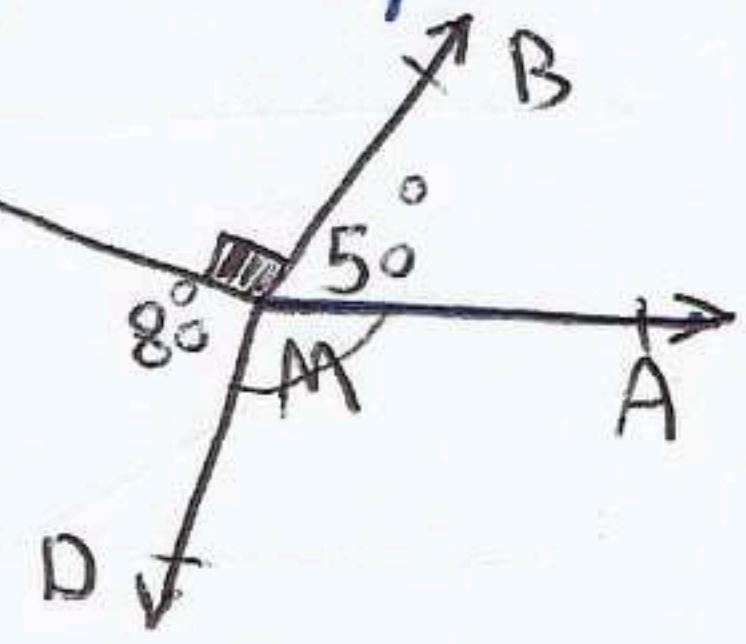


#### ④ Accumulative angles at a point

The sum of the measures of the accumulative angles at a point is  $360^\circ$

$$m(\angle AMB) + m(\angle BMC) + m(\angle CMD) + m(\angle DMA) = 360^\circ$$

$$\text{then } m(\angle DMA) = 360^\circ - (50^\circ + 90^\circ + 80^\circ) = 140^\circ$$



#### ⑤ Complementary angles

Two angles are said to be complementary if the sum of their measures is  $90^\circ$

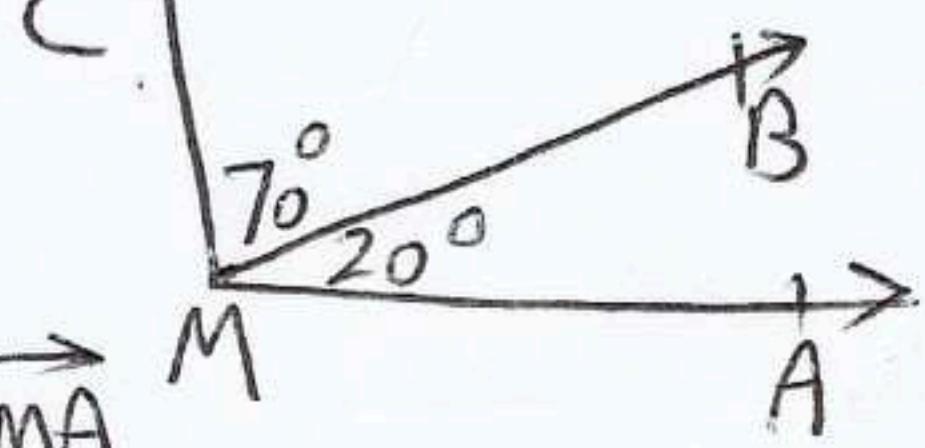
\* Two angles are either acute angles or one is zero angle and the other is a right angle.

\* The complements of the same angle are equal in measure

If  $\angle A$  complements  $\angle B$  &  $\angle C$  complement  $\angle B$  then  $m(\angle A) = m(\angle C)$

\* If the two adjacent angles are complementary angles, then their outer sides are perpendicular.

$$m(\angle AMB) + m(\angle BMC) = 90^\circ \text{ then } \overrightarrow{MC} \perp \overrightarrow{MA}$$



#### ⑥ Supplementary angles

Two angles are said to be supplementary if the sum of their measures is  $180^\circ$

\* Two angles (obtuse and acute) or (right and right) or (zero and straight angle)

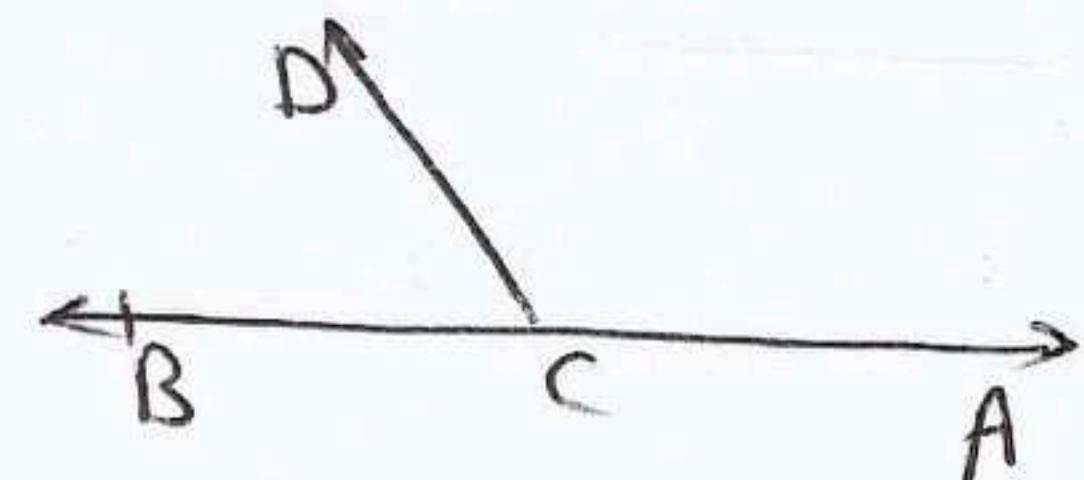
\* The supplements of the same angle are equal in measure

If  $\angle A$  supplement of  $\angle B$  and  $\angle C$  supplements  $\angle B$  then  $m(\angle A) = m(\angle C)$

\* Two adjacent angles formed by a straight line and a ray with a starting point on this straight line are supplementary.

If  $\overrightarrow{AB} \cap \overrightarrow{CD} = \{C\}$

Then  $m(\angle ACD) + m(\angle DCB) = 180^\circ$



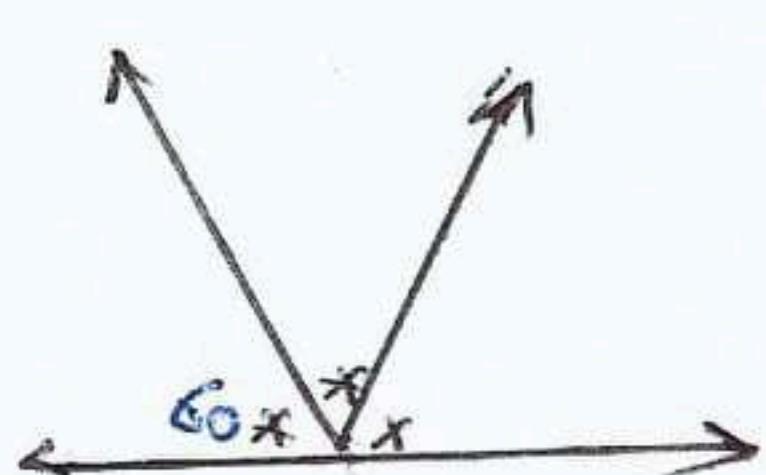
\* If two adjacent angles are supplementary, then their outer sides are on the same straight line

If  $m(\angle ACD) + m(\angle DCB) = 180^\circ$

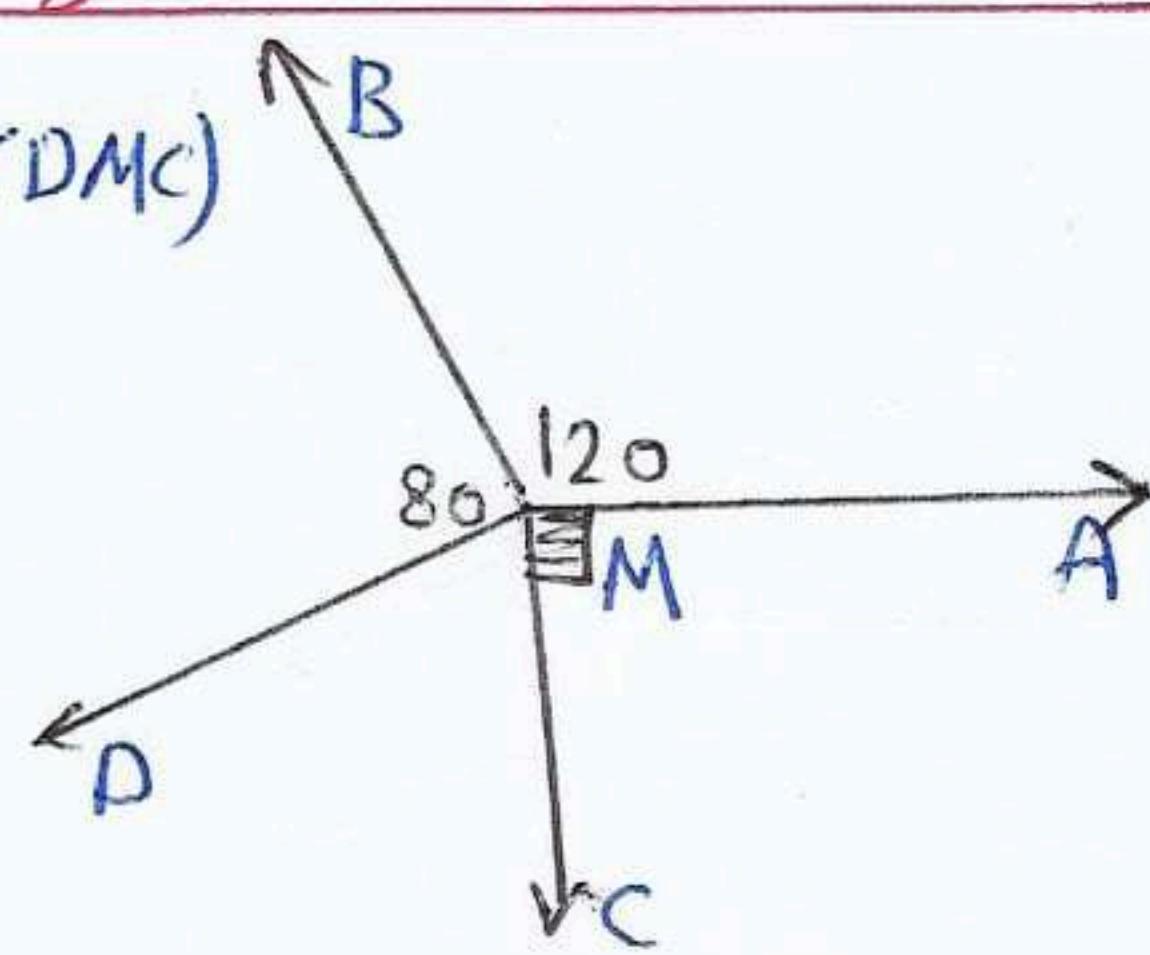
Then  $\overrightarrow{CA}$  and  $\overrightarrow{CB}$  are on the same straight line

or  $\angle ACB$  is a straight angle. try by your self.

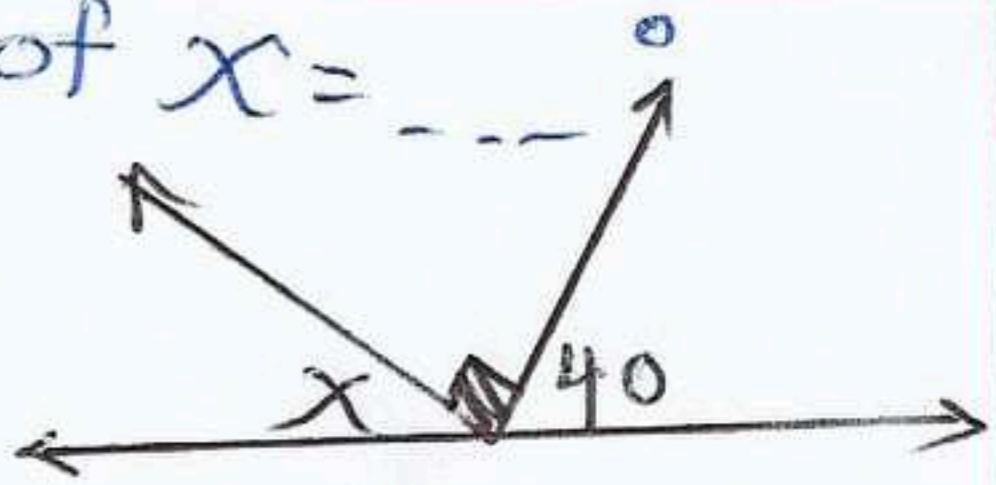
① Are  $\overrightarrow{CA}$  and  $\overrightarrow{CB}$  on same straight line? why?



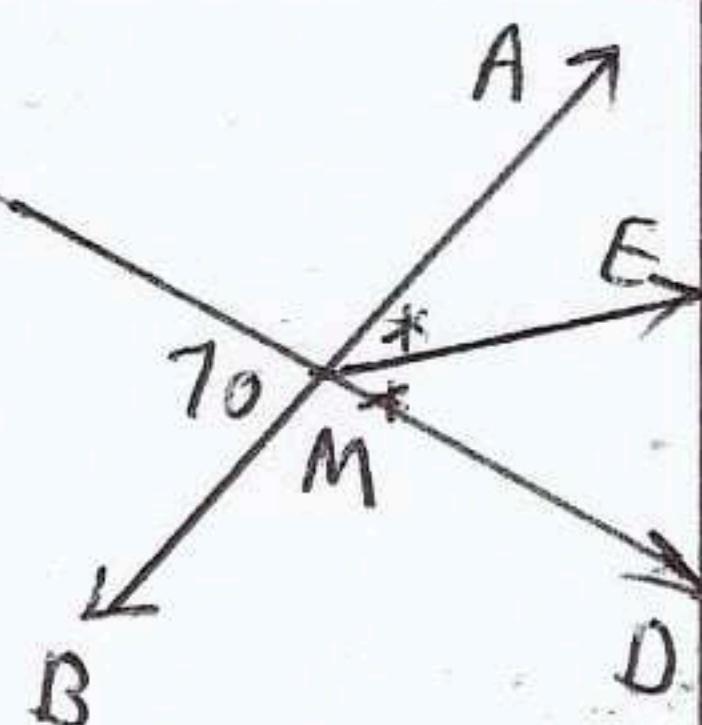
② Find  $m(\angle DMC)$



③ the value of  $x = \dots^\circ$



④  $\overrightarrow{AB} \cap \overrightarrow{DF} = \{M\}$   
find  
 $m(\angle AMF)$   
 $m(\angle AMD)$   
 $m(\angle DME)$

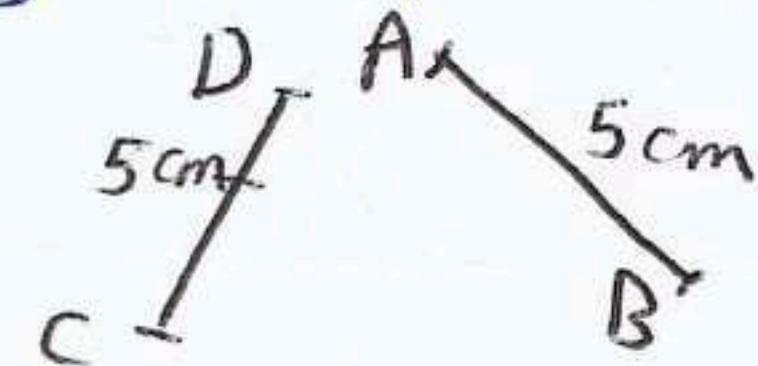


## 2] Congruence

### ① Congruence of two line segments

Two line segments are congruent if they are equal in length

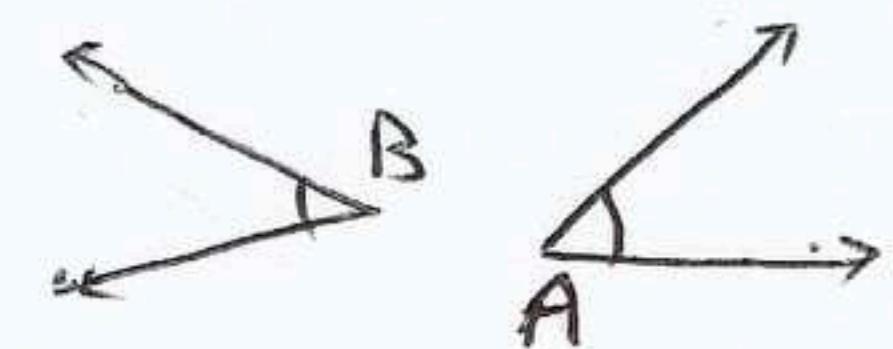
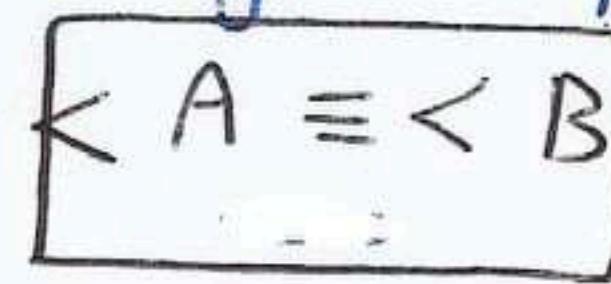
If  $AB = CD$  then  $\overline{AB} \equiv \overline{CD}$



### ② Congruence of two angles

Two angles are congruent if they are equal in measure

$m(\angle A) = m(\angle B) = 50^\circ$  then  $\angle A \equiv \angle B$

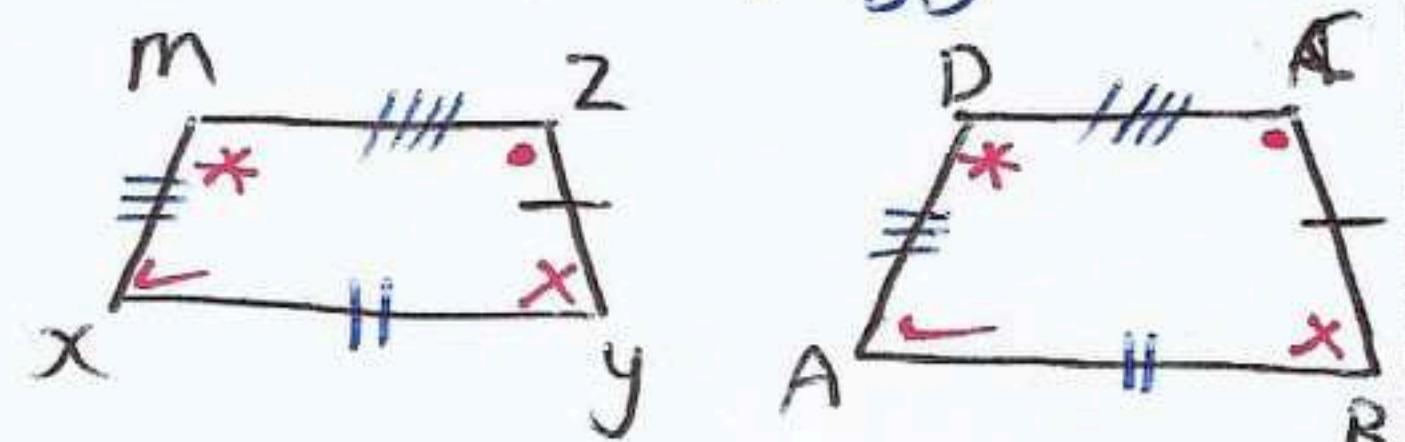


### ③ Congruence of two polygons

Two polygons are congruent if there is correspondence between their vertices such that each side and each angle in the first polygon is congruent to its corresponding element in the other polygon.

If  $m(\angle A) = m(\angle X)$ ,  $m(\angle B) = m(\angle Y)$

$m(\angle C) = m(\angle Z)$ ,  $m(\angle D) = m(\angle M)$



and  $AB = XY$ ,  $BC = YZ$ ,  $CD = ZM$ ,  $DA = MX$

then the polygon  $ABCD \equiv$  the polygon  $XYZM$

#### notes

\* The two square are congruent if their side are equal in length

\* The two rectangle are congruent if their dimension are equal.

\* The axis of symmetry of a polygon divides it into two congruent polygons

\* The diagonal of the rectangle divides its surface into two congruent triangles

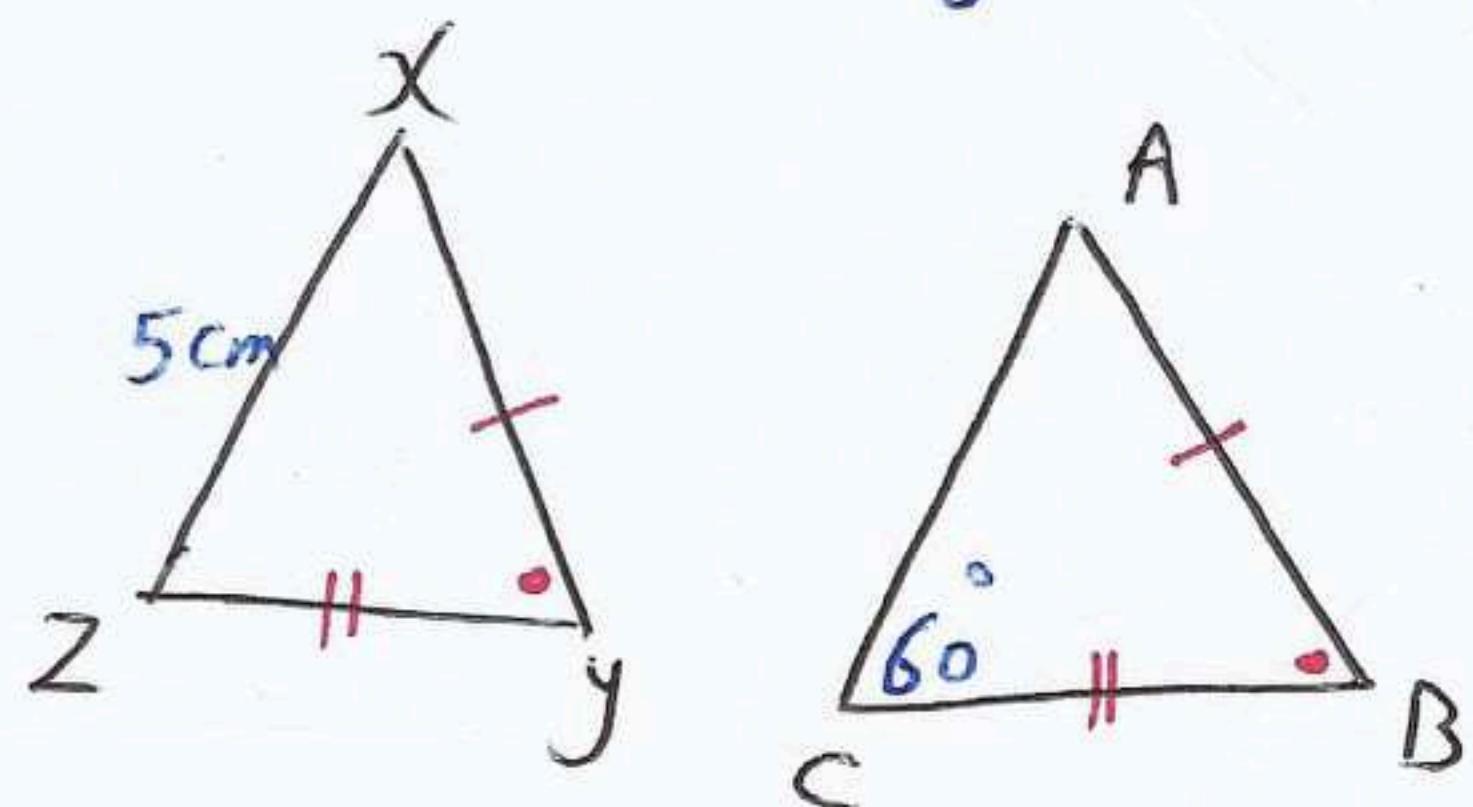
## ④ Congruent triangles

### The first case (S.A.S)

Two triangles are congruent if two sides and the included angle of one triangle are congruent to the corresponding parts of the other triangle.

In  $\triangle ABC, XYZ$

- ①  $AB = XY$  S
- ②  $m(\angle B) = m(\angle Y)$  A
- ③  $BC = YZ$  S



then  $\triangle ABC \cong \triangle XYZ$ , and we deduce that

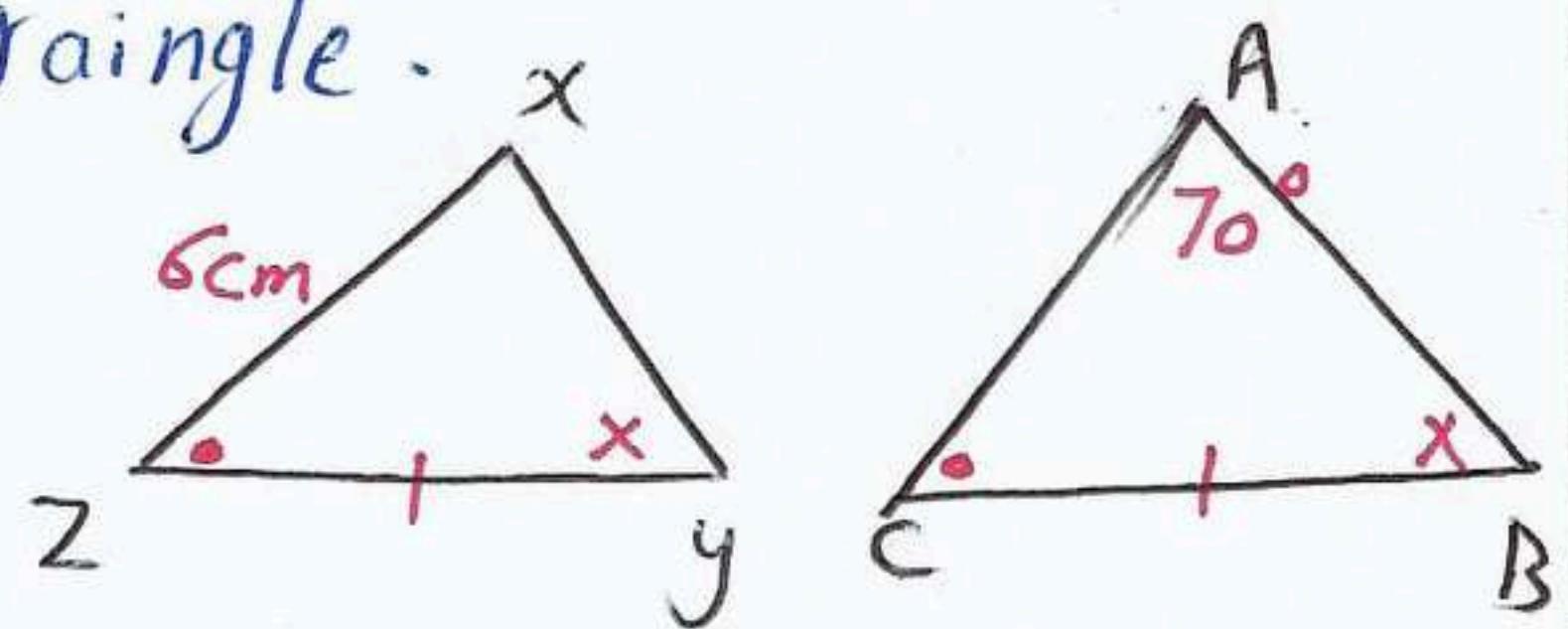
$$\textcircled{1} AC = XZ = 5\text{cm}, \textcircled{2} m(\angle C) = m(\angle Z) = 60^\circ, \textcircled{3} m(\angle A) = m(\angle X)$$

### The second case (A.S.A)

Two triangle are congruent if two angles and the side drawn between their vertices of one triangle are congruent to the corresponding parts of the other traingle.

In  $\triangle ABC, XYZ$

- ①  $m(\angle B) = m(\angle Y)$  A
- ②  $BC = YZ$  S
- ③  $m(\angle C) = m(\angle Z)$  A



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Then  $\triangle ABC \cong \triangle XYZ$  and we deduce that

$$\textcircled{1} m(\angle A) = m(\angle X) = 70^\circ, \textcircled{2} CA = ZX = 6\text{cm}, \textcircled{3} AB = XY$$

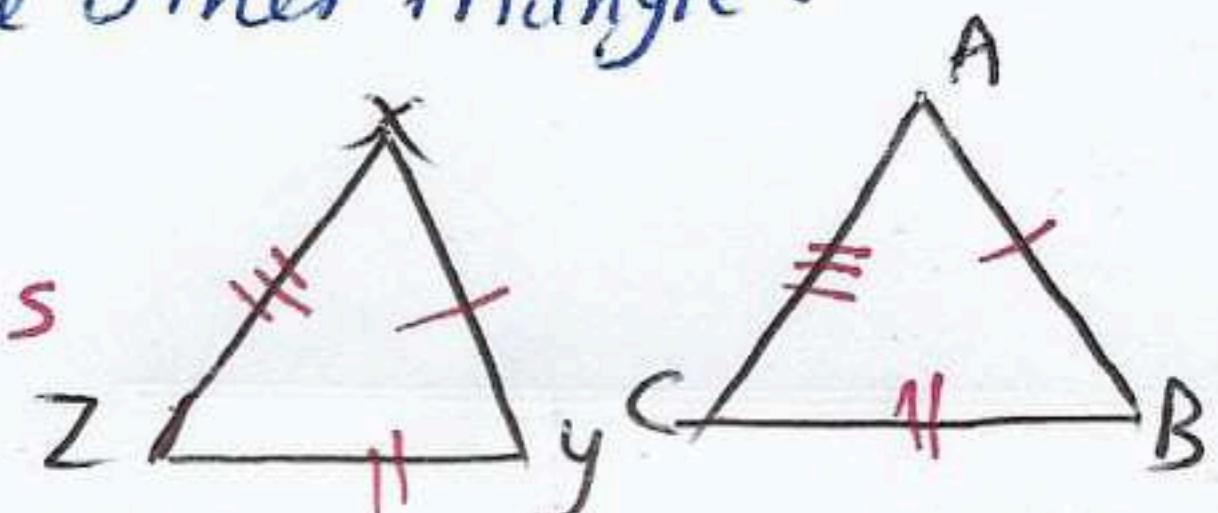
### The third case (S.S.S)

Two triangle are congruent if each side of one triangle is congruent to the corresponding side of the other triangle.

In  $\triangle ABC, XYZ$

- ①  $AB = XY$  S
- ②  $BC = YZ$  S
- ③  $AC = XZ$  S

then  $\triangle ABC \cong \triangle XYZ$

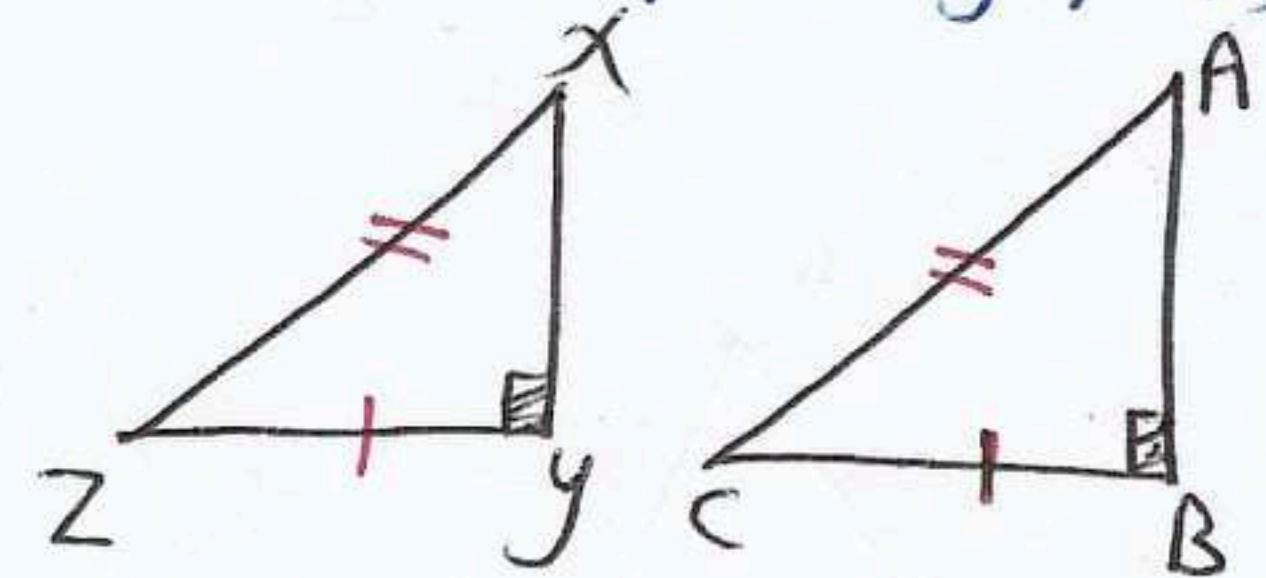


## The fourth case (R.H.S)

Two right-angled triangle are congruent if the hypotenuse and aside of one triangle are congruent to the corresponding parts of the other triangle.

In  $\triangle \Delta ABC, xyz$

- ①  $m(\angle B) = m(\angle y) = 90^\circ$  Right angle
- ②  $AC = x$  hypotenuse
- ③  $BC = y$  side



Then  $\triangle ABC \cong \triangle XYZ$

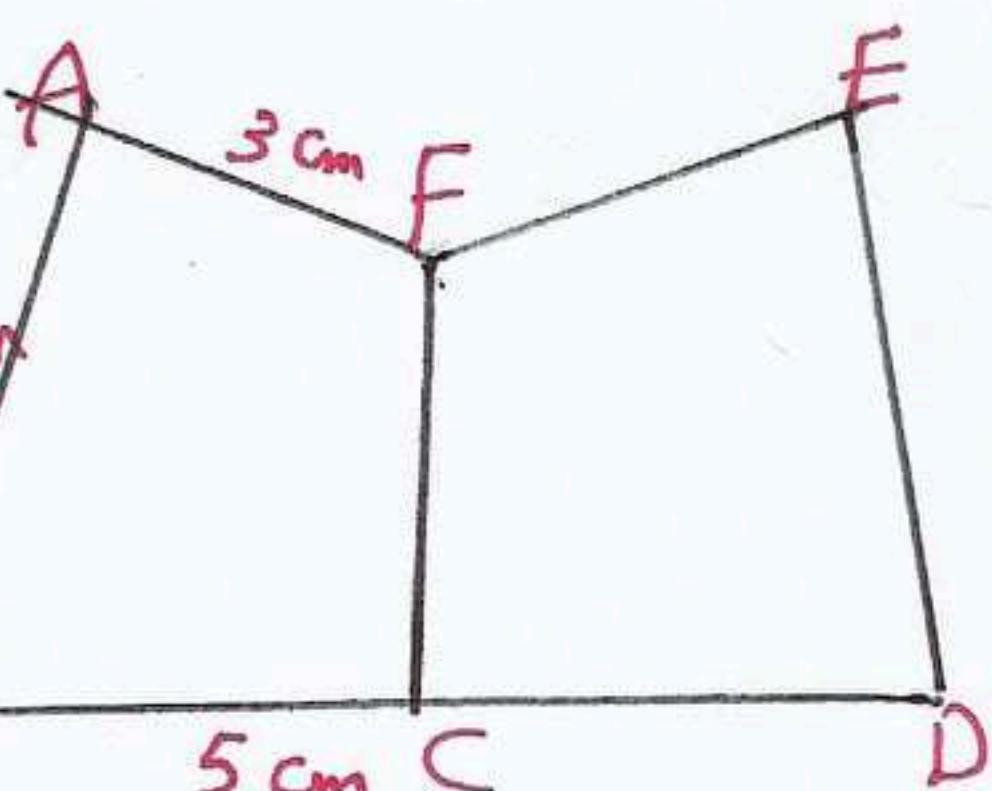
try by yourself.

① If  $C \in \overline{BD}$

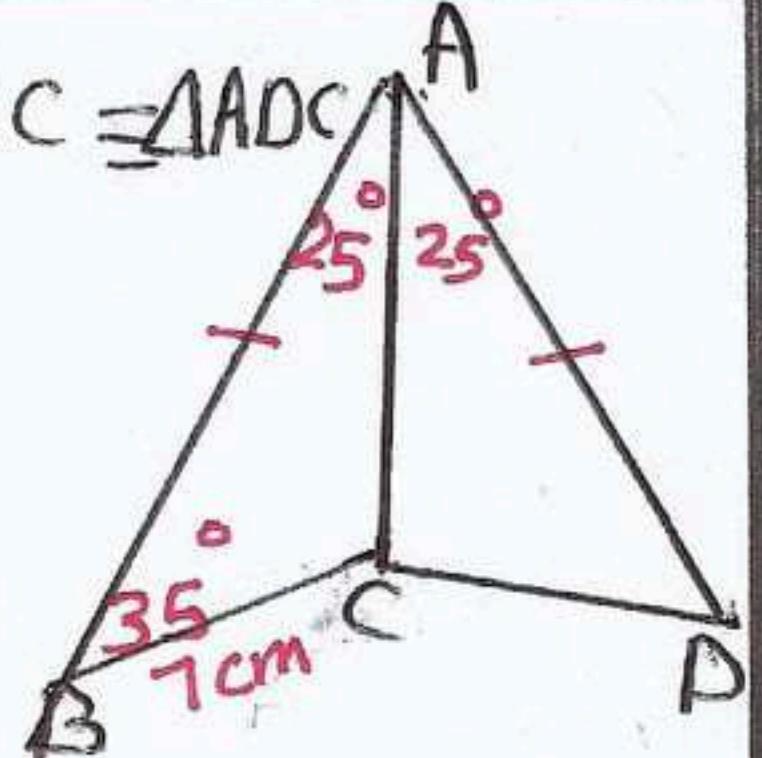
and polygon

$ABCF \cong EDCF$

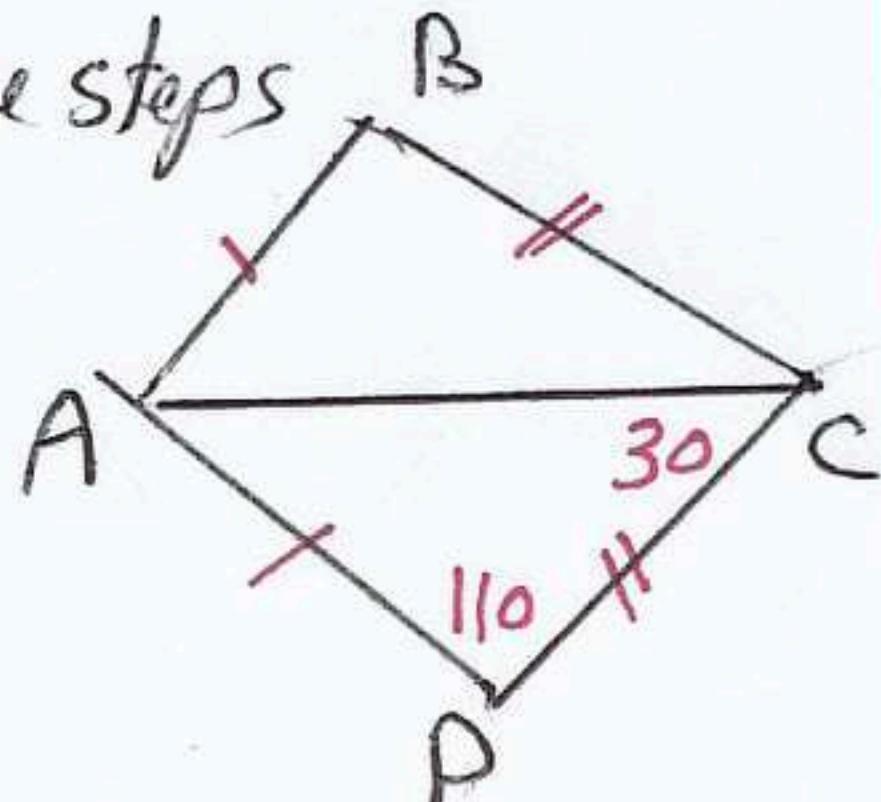
find the perimeter of figure ABDEF.



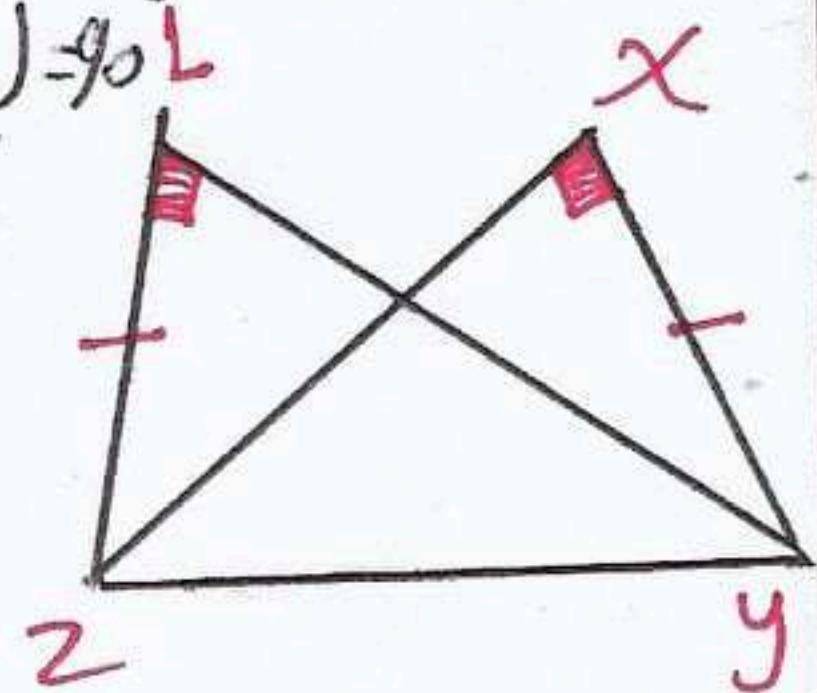
② Prove that  $\triangle ABC \cong \triangle ADC$   
find  $m(\angle D)$   
length of  $\overline{CD}$



③ Find showing the steps  
 $m(\angle ABC)$



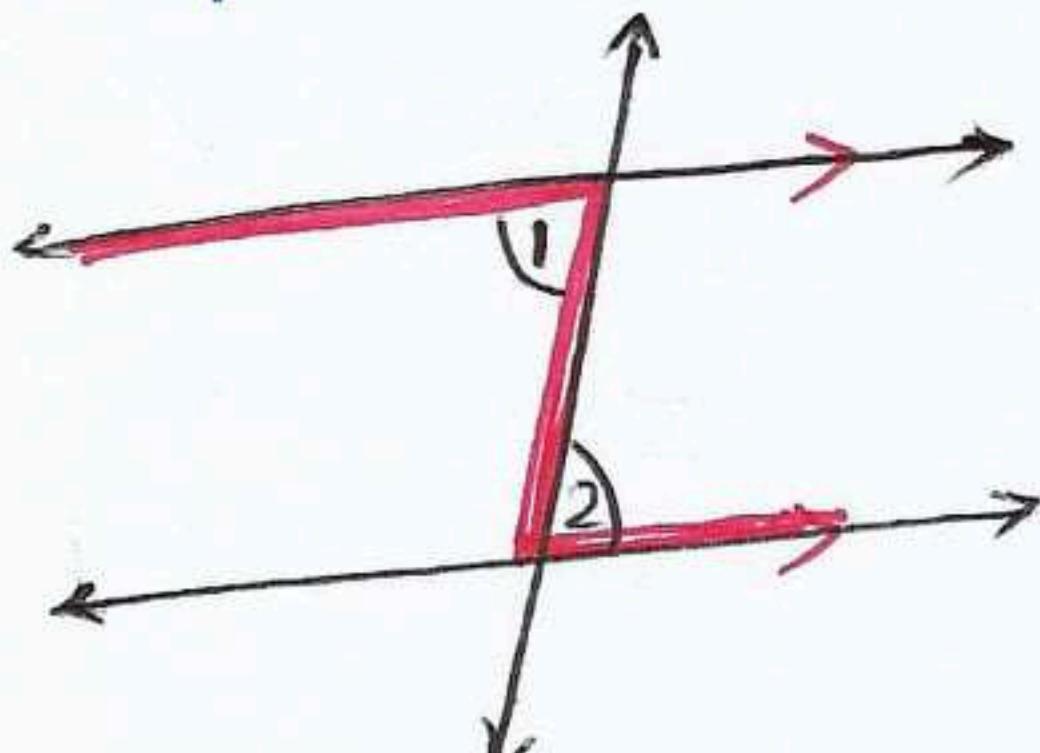
④  $m(\angle X) = m(\angle L) = 90^\circ$   
Is  $\triangle XYZ \cong \triangle LZY$   
why?



### 3 Parallelism

If a straight line intersects two parallel straight lines, then

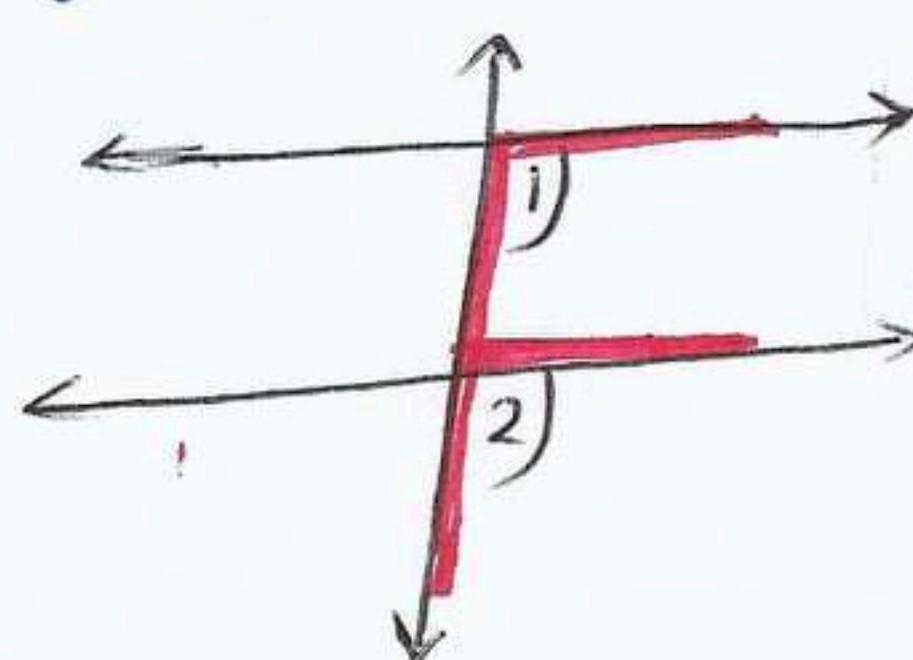
Each two alternate angles are equal in measure.  $\angle$



$$m(\angle 1) = m(\angle 2)$$

alternate angles

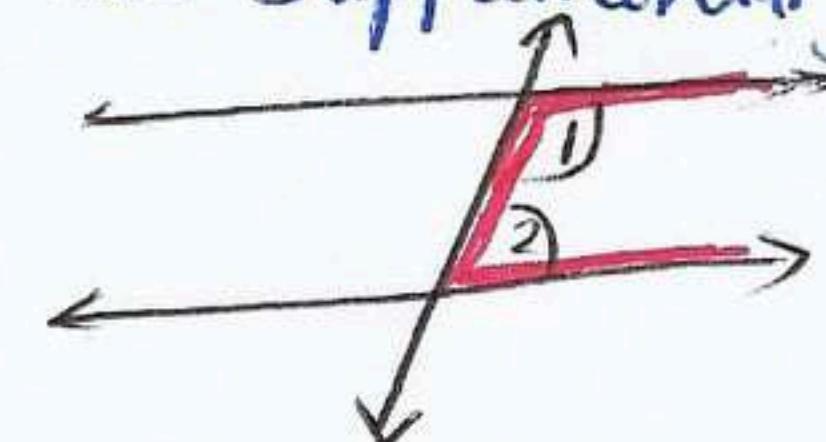
Each two corresponding angles are equal in measure.



$$m(\angle 1) = m(\angle 2)$$

corresponding angles

Each two interior angles in the same side of the transversal are supplementary.



$$m(\angle 1) + m(\angle 2) = 180^\circ$$

interior angles

How to prove the parallelism

two straight lines are parallel if a third straight line intersects them and one of the following cases is satisfied

① Two alternate angles have the same measure. or

② Two corresponding angles have the same measure. or

③ Two interior angles in the same side of the transversal are supplementary.

Geometric facts.

The perpendicular to one of two coplaner parallel straight lines is perpendicular to the other. and vice versa

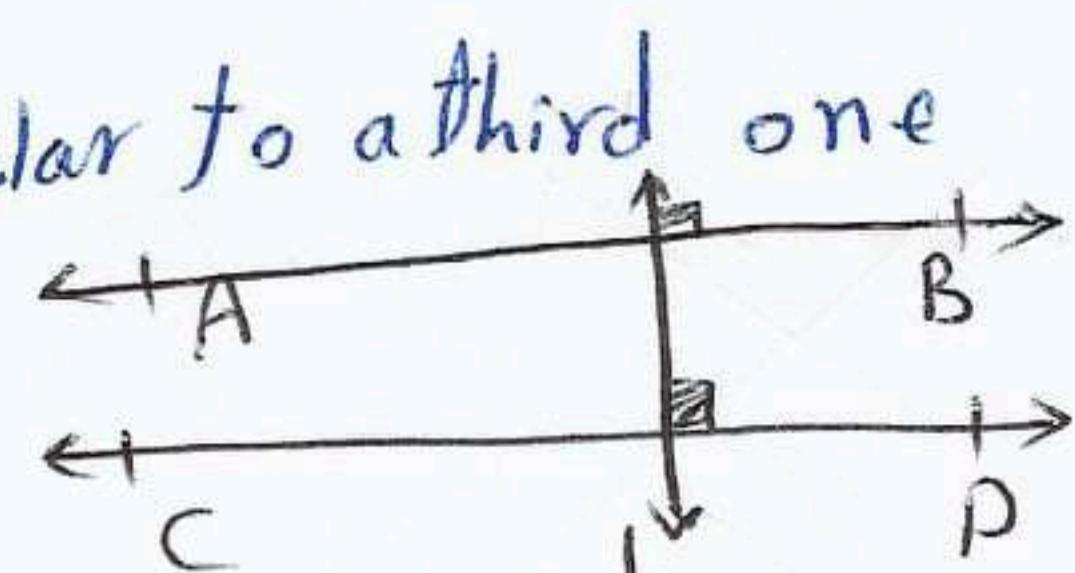
if two coplaner straight lines are perpendicular to a third one then the two straight lines are parallel.

If  $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$  and  $\overleftrightarrow{L} \perp \overleftrightarrow{AB}$

then  $\overleftrightarrow{L} \perp \overleftrightarrow{CD}$  and vice versa

If  $\overleftrightarrow{AB} \perp \overleftrightarrow{L}$  and  $\overleftrightarrow{CD} \perp \overleftrightarrow{L}$

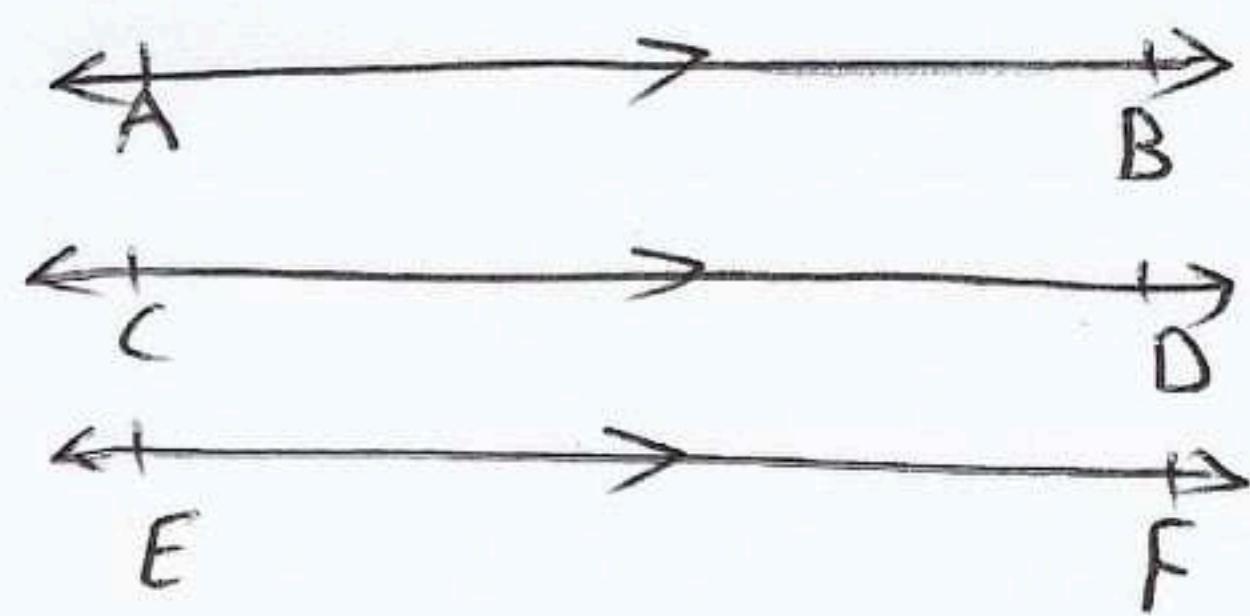
Then  $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$



**3** If two straight lines are parallel to a third, then these two straight lines are parallel

If  $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$  and  $\overleftrightarrow{EF} \parallel \overleftrightarrow{CD}$

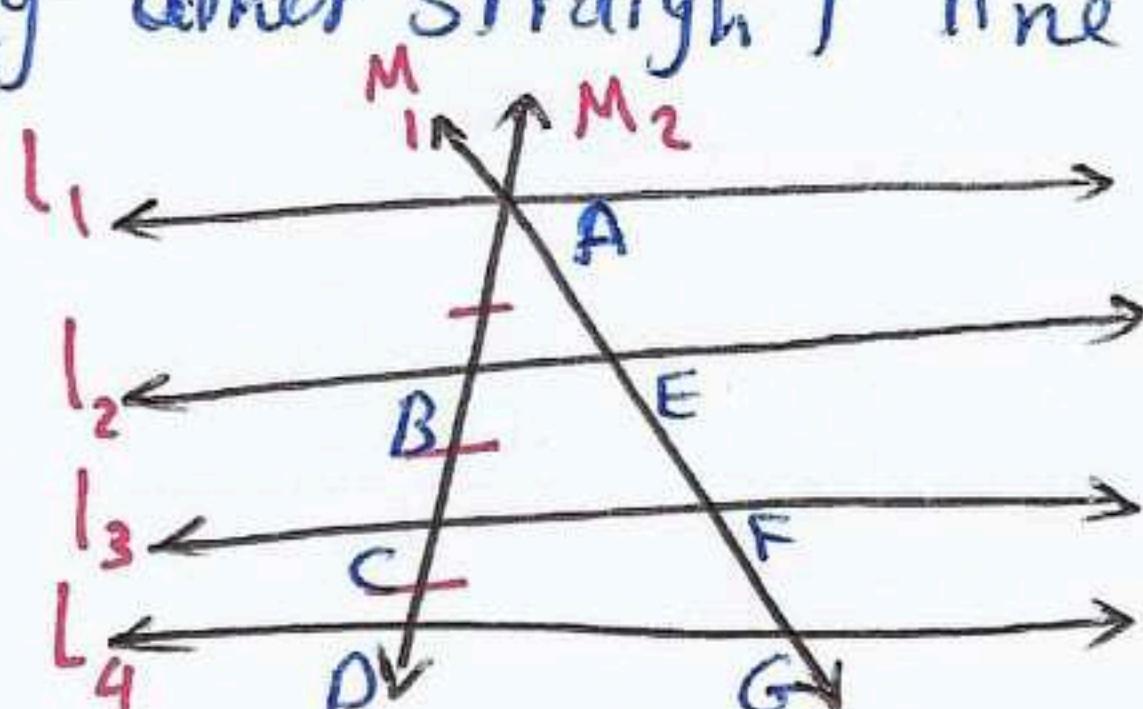
Then  $\overleftrightarrow{AB} \parallel \overleftrightarrow{EF}$



**4** If parallel straight lines divide a straight line into segments

of equal lengths, then they divide any other straight line into segments of equal lengths.

If  $\overleftrightarrow{l_1} \parallel \overleftrightarrow{l_2} \parallel \overleftrightarrow{l_3} \parallel \overleftrightarrow{l_4}$  and  $AB = BC = CD$

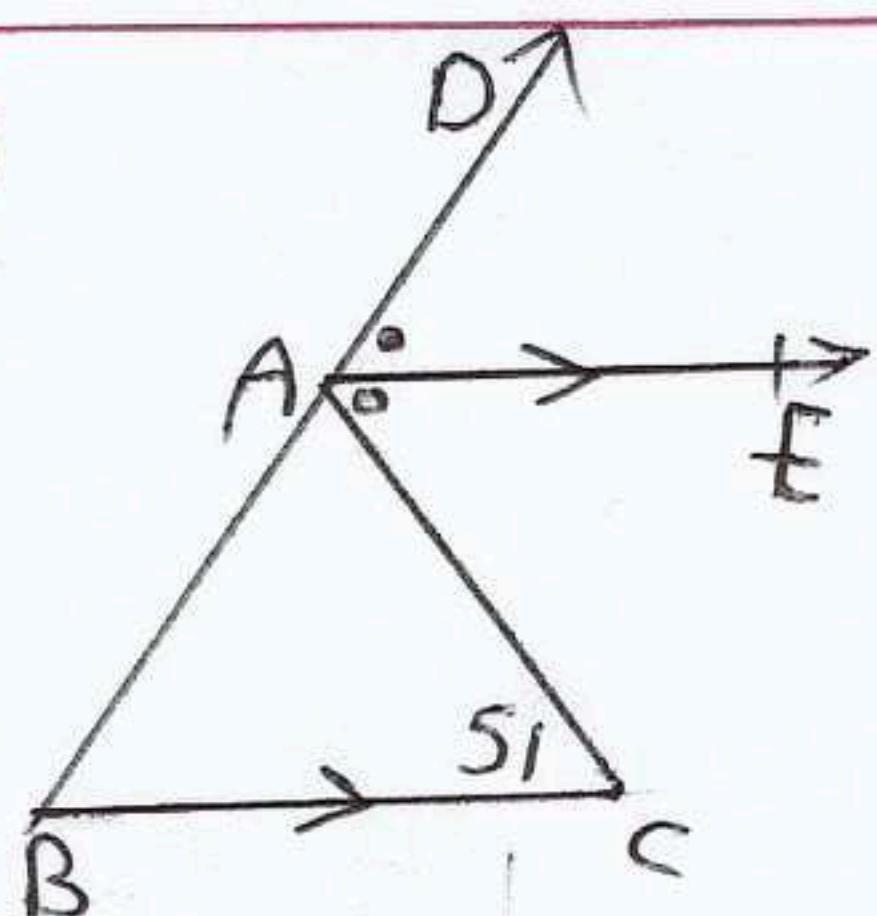


Then  $AE = EF = FG$

Try by yourself

① Find  $m(\angle CAE)$

$m(\angle B)$



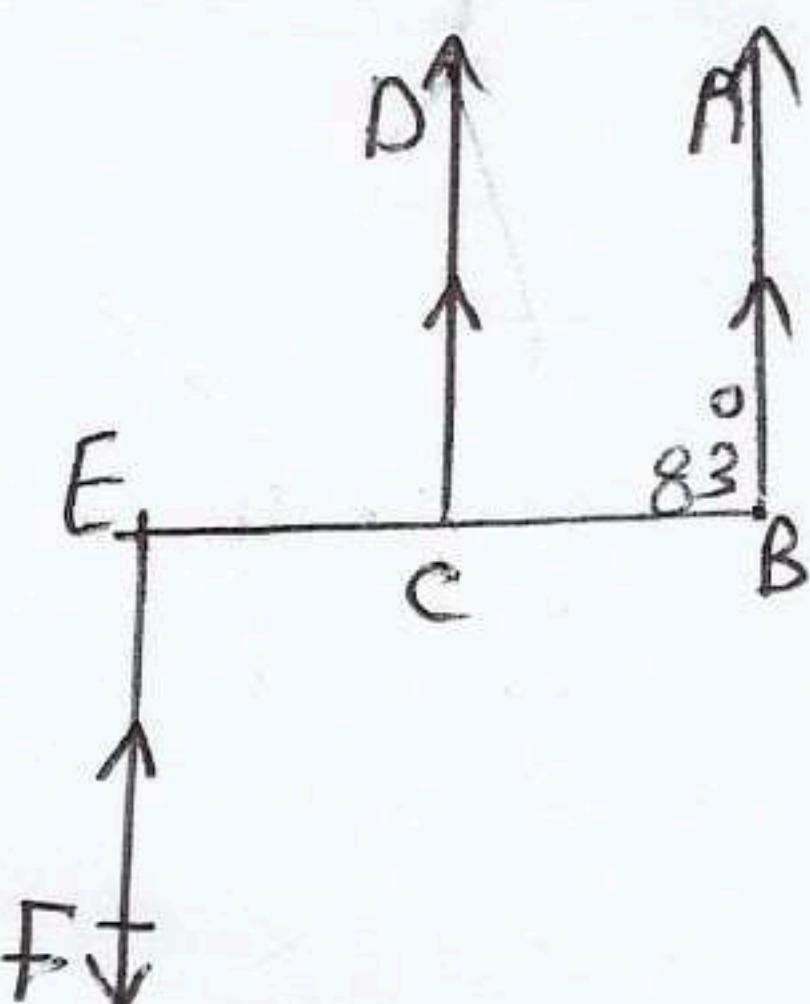
②  $y \in \overleftrightarrow{zx}$   $150^\circ$

Find  $m(\angle YZ)$

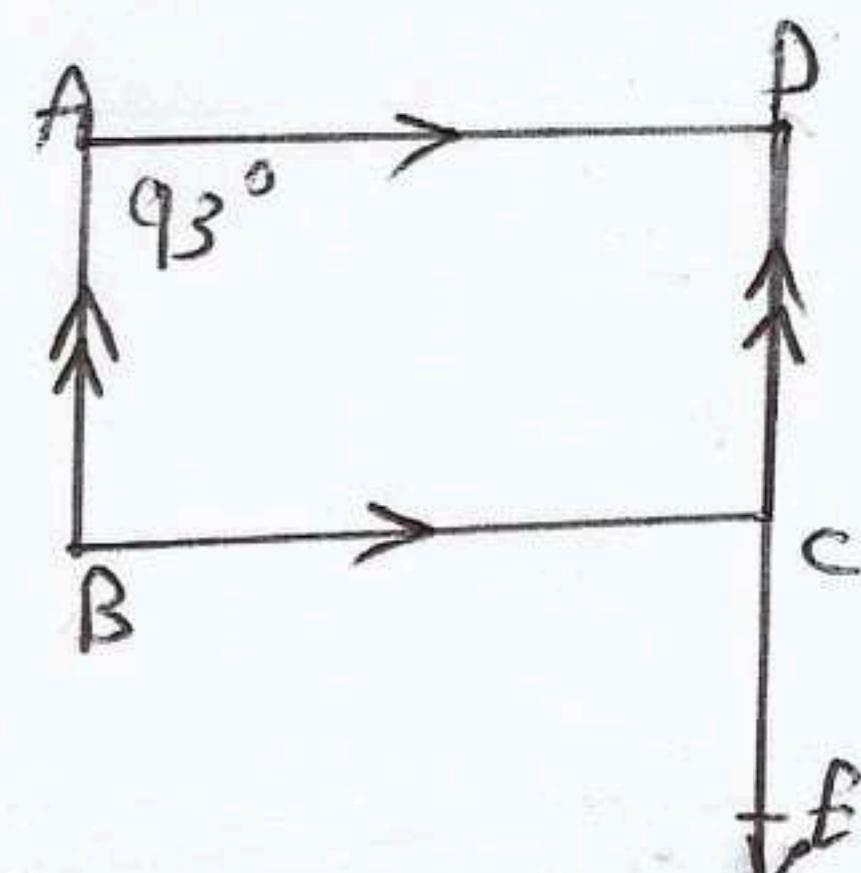
IS  $\overleftrightarrow{YM} \parallel \overleftrightarrow{ZE}$ ? why?  $30^\circ$



③ Find  $m(\angle CEF)$   
 $m(\angle DCE)$



④ Find  
 $m(\angle BCE)$   
 $m(\angle D)$



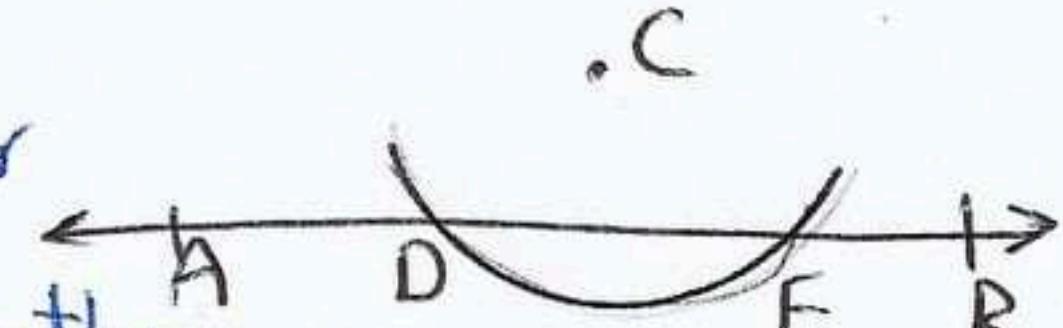
## 4] Geometric constructions

① Drawing a perpendicular from a point outside a straight line:

- ① Using the compasses at C as a centre and with a suitable radius draw an arc to intersect  $\overleftrightarrow{AB}$  at the two points D and E



- ② At D and E as a centre with a radius greater than  $\frac{1}{2} DE$  draw two arcs to intersect each other at L

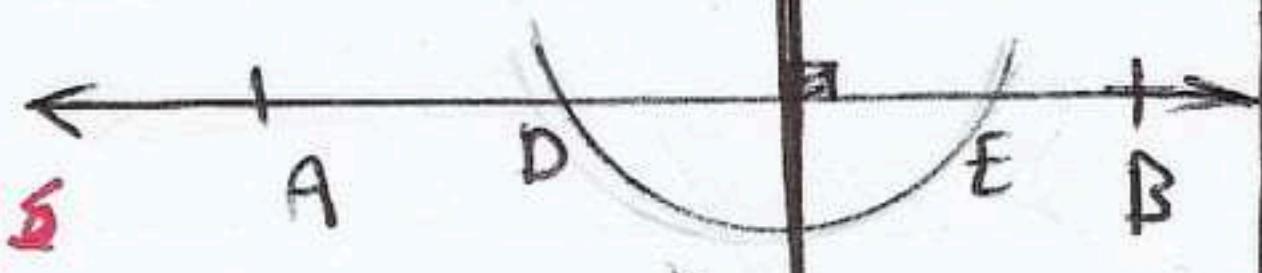


- ③ Draw  $\overleftrightarrow{CL}$

② Drawing a perpendicular from a point belonging to the straight line

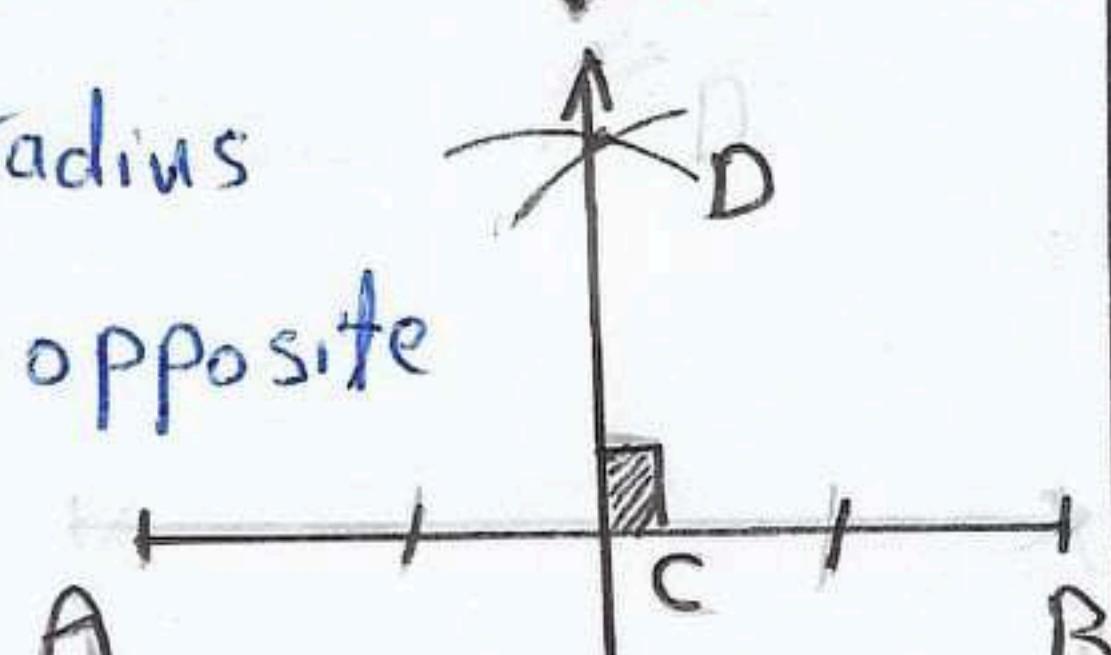
$\overleftrightarrow{AB}$  is given and  $C \in \overleftrightarrow{AB}$

We do the same steps



③ Bisecting a given line segment or drawing the axis of symmetry

- ① Using the compasses at A as a centre with a radius greater than  $\frac{1}{2} AB$  draw two arcs in the opposite side of  $\overline{AB}$



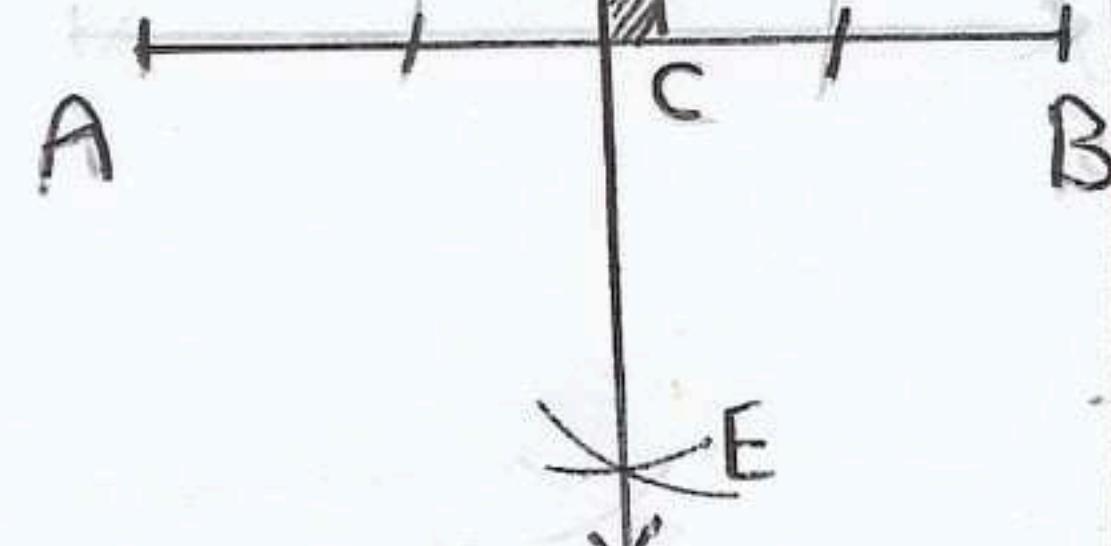
- ② Do the same step at B

- ③ Draw  $\overleftrightarrow{DE}$  to cut  $\overline{AB}$  at C

C is the mid point of  $\overline{AB}$  and  $\overleftrightarrow{DE} \perp \overline{AB}$

The axis of symmetry of a line segment.

It is the straight line perpendicular to it from its midpoint



#### ④ Constructing the bisector of a given angle.

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- ① Taking B as a center, draw an arc intersect  $\overrightarrow{BA}$  and  $\overrightarrow{BC}$  at D and E
- ② taking D and E as centres draw two arcs intersect at the point X
- ③ Draw  $\overrightarrow{BX}$  to be the bisector of  $\angle ABC$

#### ⑤ Constructing an angle to be congruent to a given angle

$\angle ABC$  is a given angle

- ① Draw  $\overrightarrow{YL}$  as one side of the angle

- ② Taking B as a center draw an arc intersect  $\overrightarrow{BA}$  and  $\overrightarrow{BC}$  at D and E

- ③ with y as a centre and with the same radius draw an arc cut  $\overrightarrow{YL}$  at X

- ④ with X as a centre and with radius equal the length of DE draw another arc to cut the previous arc at Z

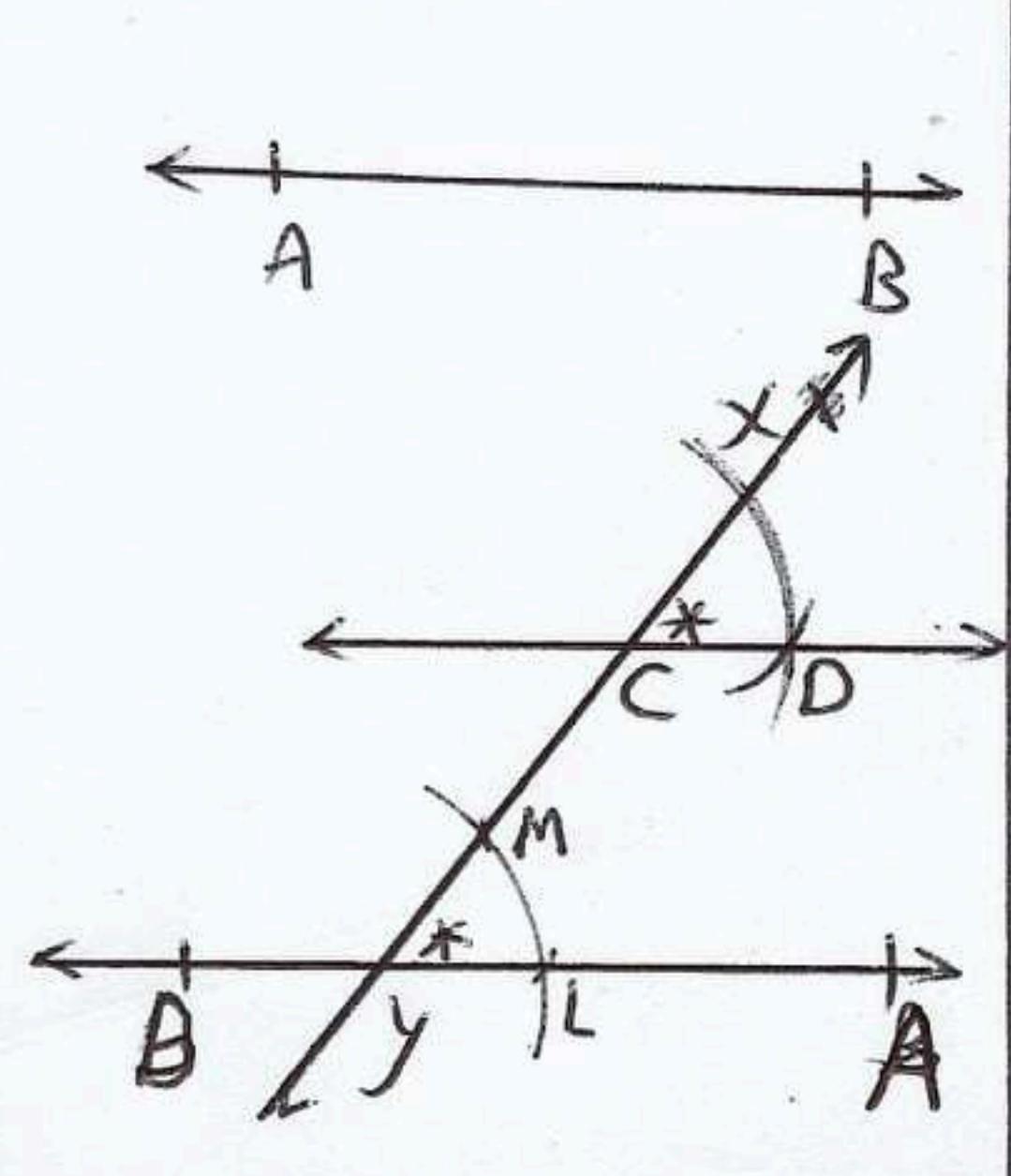
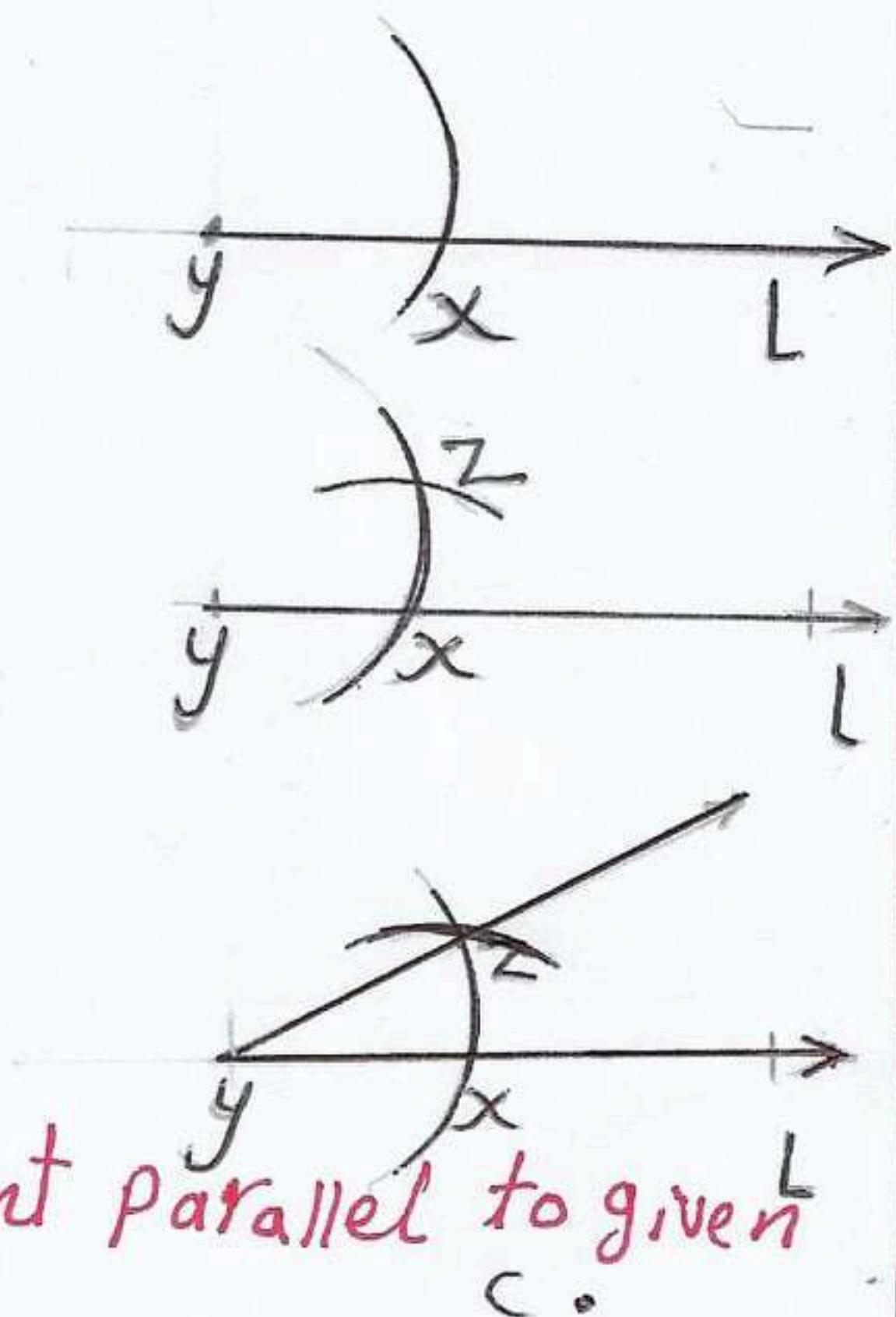
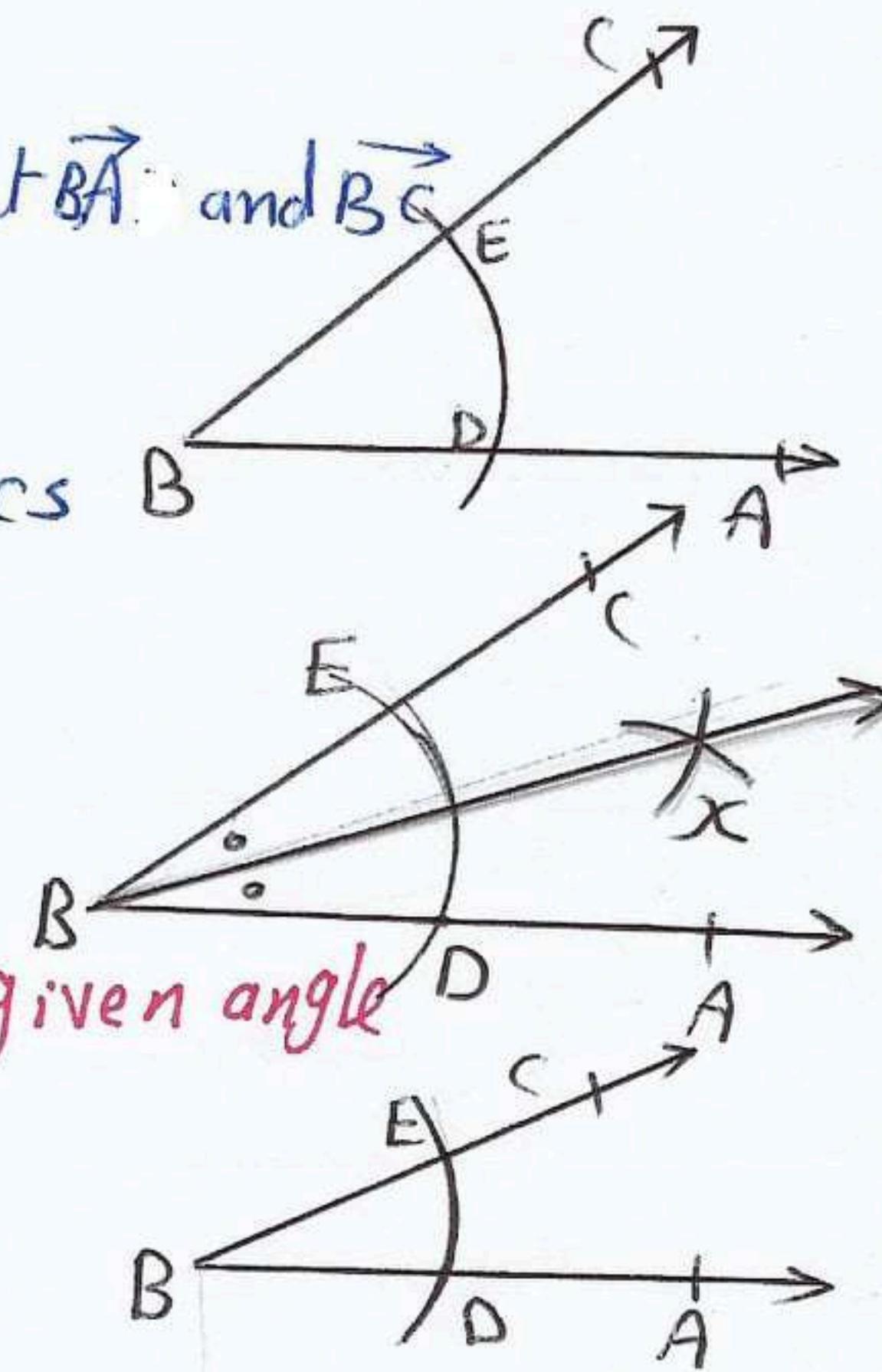
- ⑤ draw  $\overrightarrow{YZ}$

- ⑥ Drawing a straight line from a given point parallel to given straight line.

$\overleftrightarrow{AB}$  is a given straight line and  $C \notin \overleftrightarrow{AB}$

- ① draw  $\overleftrightarrow{XY}$  passing through the point C and cutting  $\overleftrightarrow{AB}$  at Y

- ② draw at C the angle  $XC'D$  corresponding to  $\angle AYX$  &  $\angle XCD \equiv \angle XYA$  using the previous construction then  $\overleftrightarrow{CD} \parallel \overleftrightarrow{AB}$



## Model 1

**1** Complete each of the following:

① The perpendicular bisector of a line segment is called ..... axis of symmetry.

② If  $\triangle ABC \cong \triangle XYZ$ ,  $m(\angle A) + m(\angle B) = 140^\circ$  then  $m(\angle Z) = \dots$   
 $m(\angle C) = 180 - 140 = 40^\circ$   
 since  $\triangle ABC \cong \triangle XYZ$  then  $m(\angle Z) = m(\angle C) = 40^\circ$

③ If  $m(\angle B) = 105^\circ$ , then  $m(\text{reflex } \angle B) = \dots$

$$m(\text{reflex } \angle B) = 360 - 105^\circ = 255^\circ$$

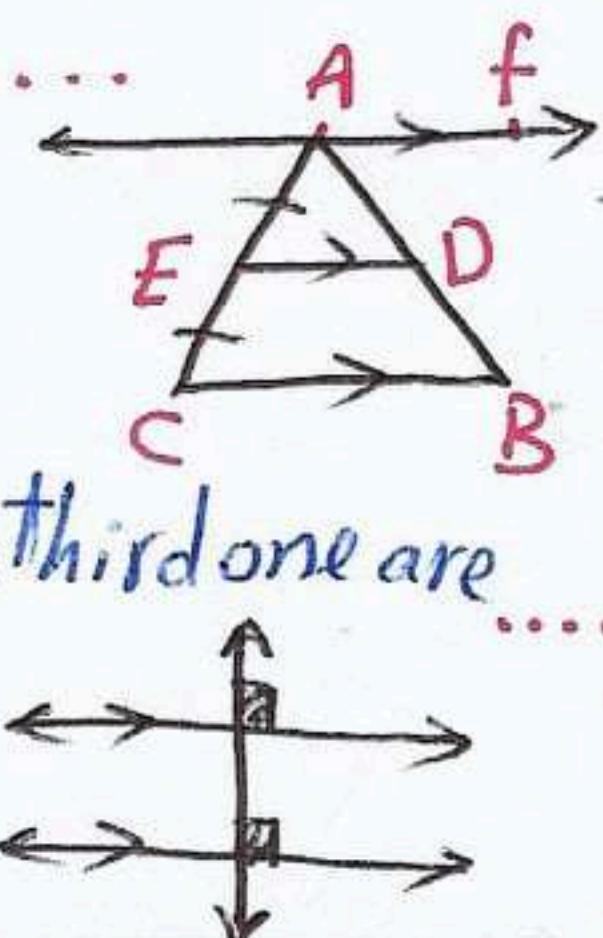
④ If  $\overrightarrow{MB} \cap \widehat{AC} = \{m\}$ ,  $m(\angle AMB) = 60^\circ$ , then the value of  $x = \dots$   
 $3x = 180^\circ - 60^\circ = 120^\circ$ , then  $x = \frac{120}{3} = 40^\circ$

⑤ Two right-angled triangles are congruent if .....

the hypotenuse and aside of one triangle are congruent to the corresponding parts of the other triangle.

⑥ If  $\angle x \equiv \angle y$ ,  $\angle x$  and  $\angle y$  are supplementary angles, then  $m(x) = \dots$   
 $m(\angle x) + m(\angle y) = 180^\circ$  then  $m(\angle x) = m(\angle y) = \frac{180}{2} = 90^\circ$

⑦  $\overleftrightarrow{AF} \parallel \overleftrightarrow{DE} \parallel \overleftrightarrow{BC}$ ,  $AE = EC$ , then  $AD:AB = \dots : \dots$   
 $AD = DB$  then  $AD:AB = 1:2$



⑧ The two straight lines that are perpendicular to a third one are ..... parallel

⑨ The measure of each of two equal complementary angles =  $\dots$   
 $m(\angle x) + m(\angle y) = 90^\circ \Rightarrow m(\angle x) = \frac{90}{2} = 45^\circ$

⑩ If two straight lines intersect, then each two..... angles have the same measure vertically opposite

⑪ If  $\Delta ABC \cong \Delta LMN$ , then  $m(\angle ACB) = m(\angle ....)$   
 $m(\angle ACB) = m(\angle LNM)$

2] In the opposite figure:

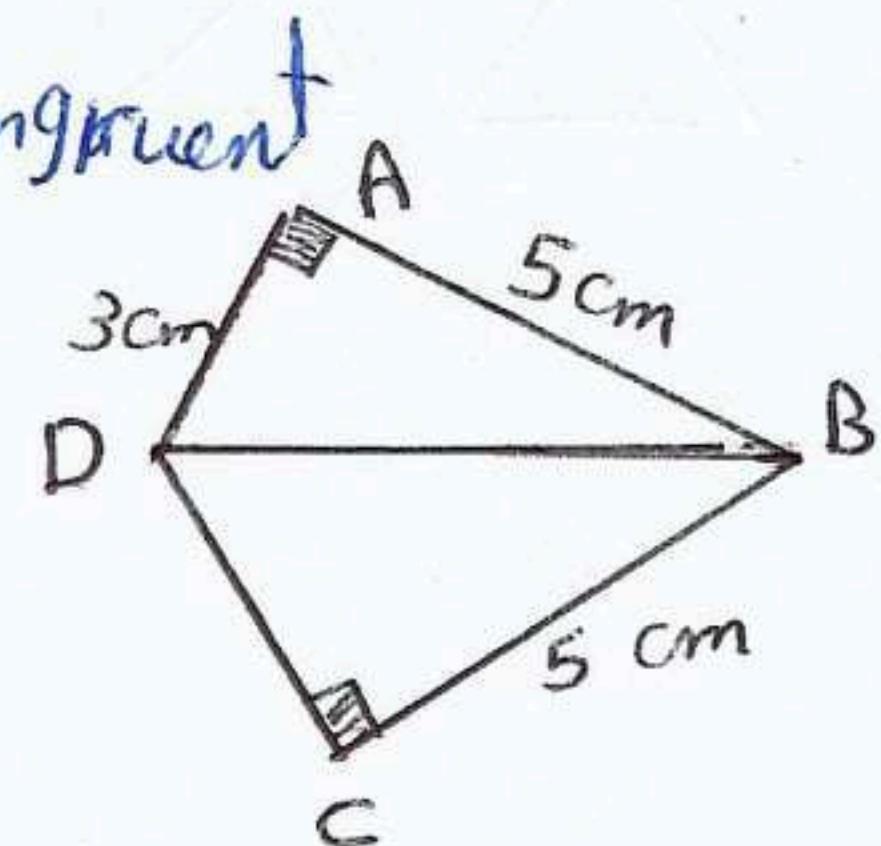
mention the conditions for  $\Delta ABD, \Delta CBD$  to be Congruent

then find the length of  $\overline{CD}$

Solution

In  $\Delta ABD$  and  $\Delta CBD$ ,

- ①  $m(\angle C) = m(\angle A) = 90^\circ$  R
- ②  $\overline{BD}$  is a common hypotenuse H
- AB = CB S



Then  $\Delta ABD \cong \Delta CBD$

Then  $CD = AD = 3\text{cm}$

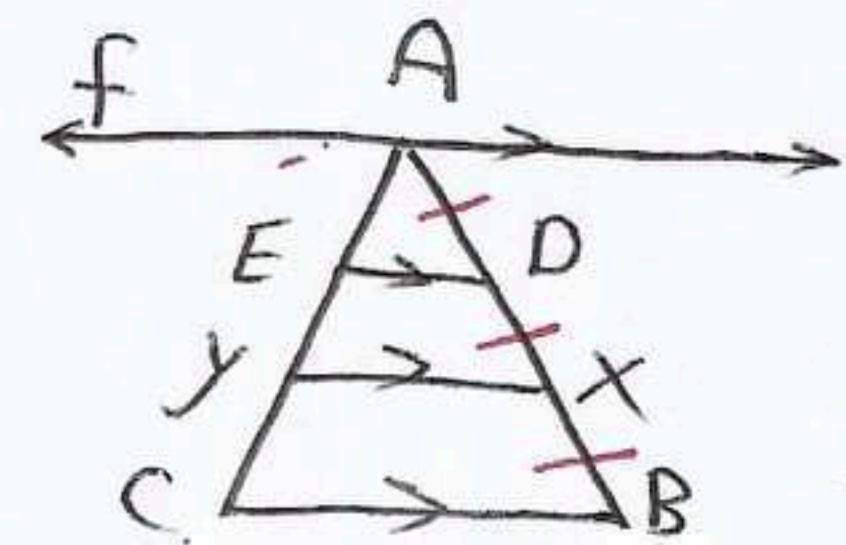
⑥ in the opposite figure.  $AC = 9\text{cm}$

find the length of  $\overline{Ay}$  give the reason

Since  $\overrightarrow{Af} \parallel \overrightarrow{DE} \parallel \overrightarrow{xy} \parallel \overrightarrow{BC}$

and  $AD = Dx = xB$

Then  $AE = Ey = yC = \frac{9}{3} = 3\text{cm}$ , then  $Ay = 3+3 = 6\text{cm}$



④ In the opposite figure

Find  $m(\angle ACE)$

Solution

Since  $\overrightarrow{AB} \parallel \overrightarrow{CD}$

Then  $m(\angle A) = m(\angle ACD) = 45^\circ$

(alternate angles)

Since  $\overrightarrow{CD} \parallel \overrightarrow{EF}$

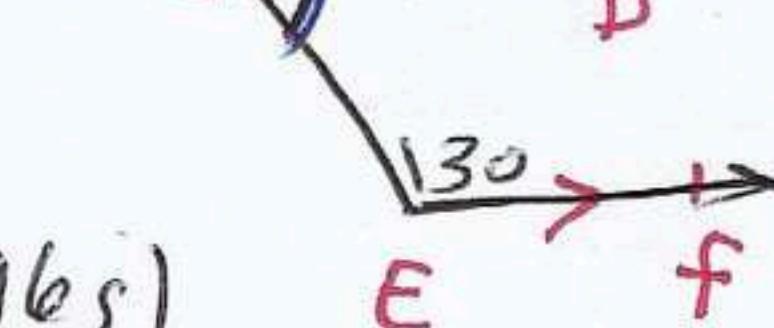
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Then  $m(\angle DCE) + m(\angle E) = 180^\circ$

$$m(\angle DCE) = 180 - 130 = 50^\circ$$

interior angles

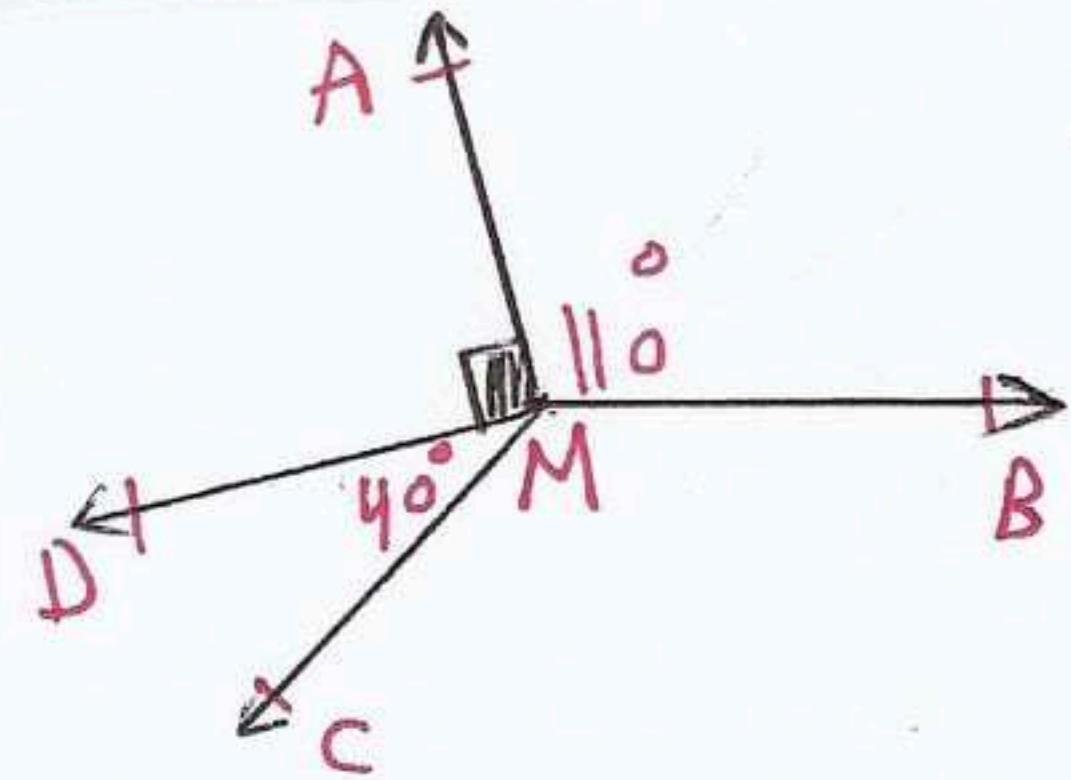
$$\text{, then } m(\angle ACE) = \frac{45+50}{2} = 47.5^\circ$$



## ⑥ In the opposite figure

Find with steps  $m(\angle BMC)$

Solution



Since sum of measures of accumulative angles at point M is  $360^\circ$

Then  $m(\angle BMC) = 360^\circ - (110^\circ + 90^\circ + 40^\circ) = 120^\circ$

## ⑤@ In the opposite figure

write the conditions for  $\triangle AMB \cong \triangle DMC$  to be Congruent

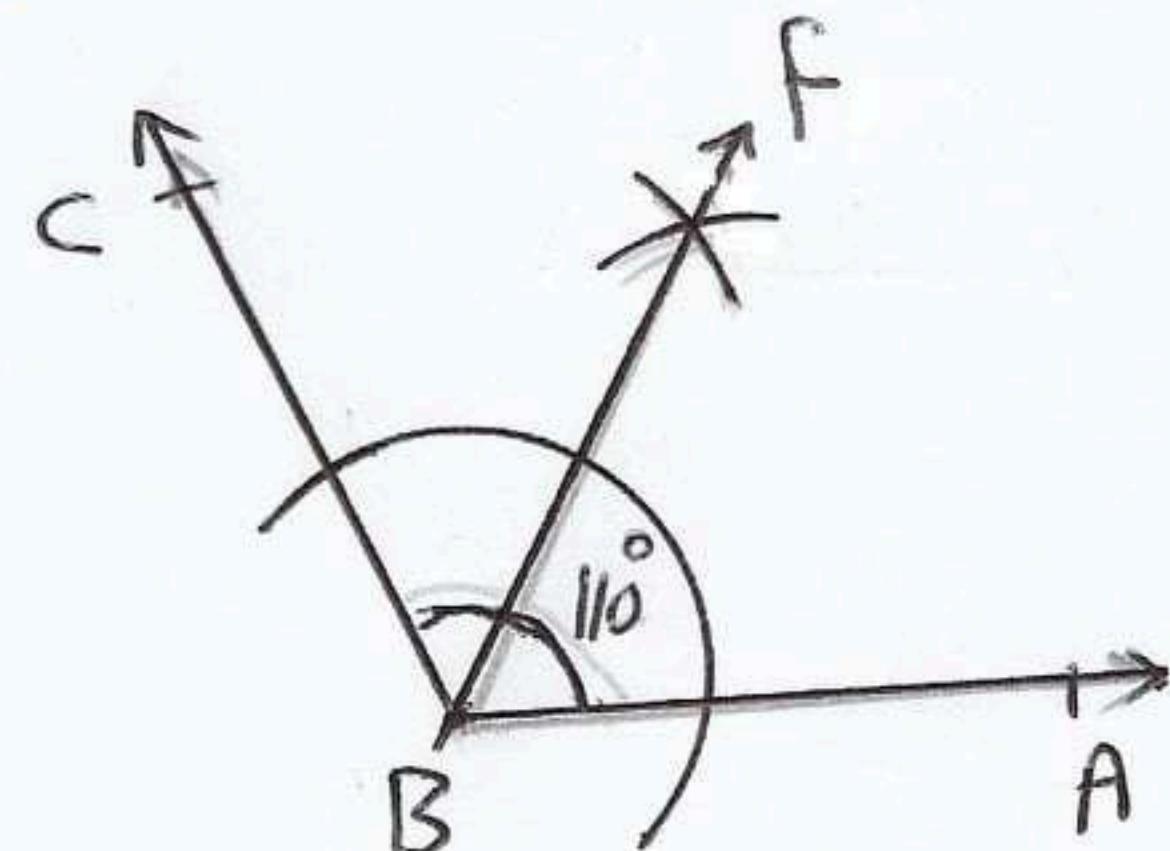
In  $\triangle AMB, \triangle DMC$

Solution

$$\begin{cases} AM = DM \\ m(\angle AMB) = m(\angle DMC) \quad (\text{V.O.A}) \\ BM = CM \end{cases}$$

Then  $\triangle AMB \cong \triangle DMC$

(b) Using the geometric instruments , draw  $\angle ABC$  of measure  $110^\circ$  , then draw  $\overrightarrow{BF}$  to bisect the angle .



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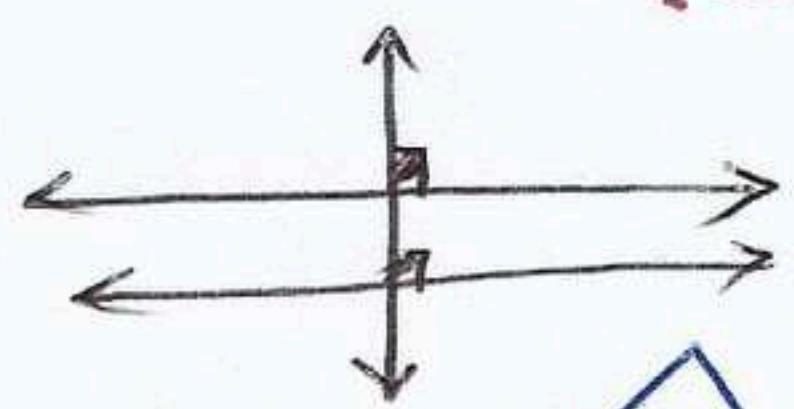
① Complete each of the following **Model 2**

- ① The sum of the measures of the accumulative angles at a point =  $360^\circ$
- ② If a straight line intersects two parallel straight lines, then each two corresponding angles are equal in measure.
- ③ If  $m(\angle A) = 110^\circ$  then  $m(\text{reflex } \angle A) = \dots \dots$   
 $360 - 110 = 250^\circ$
- ④ Two right angled triangles are congruent if hypotenuse and a side of one triangle are congruent to the corresponding parts of the other triangle
- ⑤ The two adjacent angles formed by the intersection of a straight line and a ray are supplementary
- ⑥ If  $\angle x$  complements  $\angle y$  and  $\angle x \equiv \angle y$  then  $m(\angle x) = \dots \dots$   
 $m(\angle x) + m(\angle y) = 90^\circ \Rightarrow m(\angle x) = m(\angle y) = \frac{90}{2} = 45^\circ$
- ⑦ The number of triangles in the figure  equals 8
- ⑧ If the ratio between the measures of two supplementary angles is  $5:13$ , then the measure of the smaller angles equals ...  
 $1^{\text{st}} \text{ angle : } 2^{\text{nd}} \text{ angle : sum}$       supplementary  $\Rightarrow 1^{\text{st}} \text{ angle} + 2^{\text{nd}} \text{ angle} = 180^\circ$   
 $5 : 13 : 18$   
 $? : : 180 \Rightarrow \text{smaller angles} = \frac{180 \times 5}{18} = 50^\circ$
- ⑨  $\triangle ABC \cong \triangle XYZ$ ,  $m(\angle A) + m(\angle B) = 100^\circ$  then  $m(\angle Z) = \dots \dots$

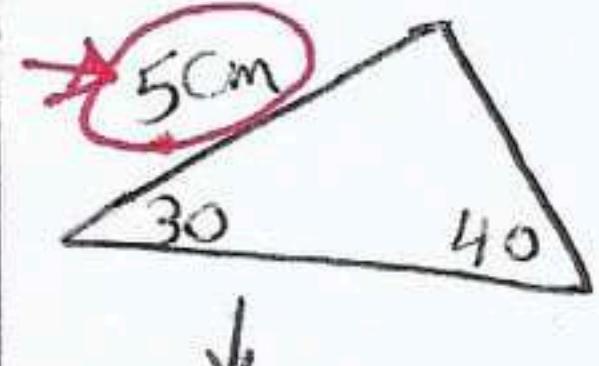
$$m(\angle Z) = m(\angle C) = 180 - 100 = 80^\circ$$

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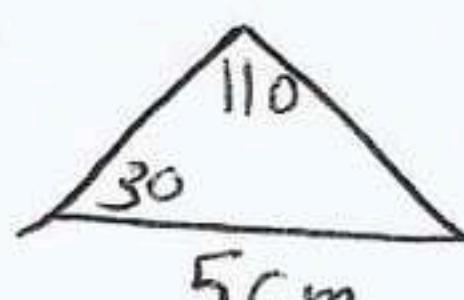
⑩ The two straight lines that are perpendicular to a third one are parallel



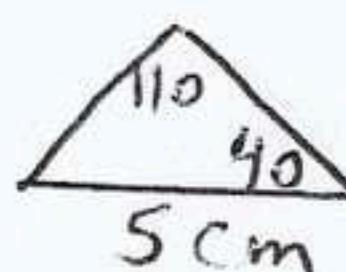
⑪ The figure is not congruent to the opposite figure



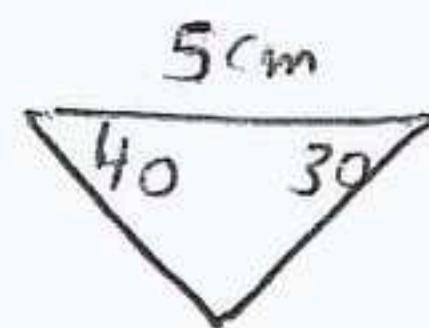
not congruent



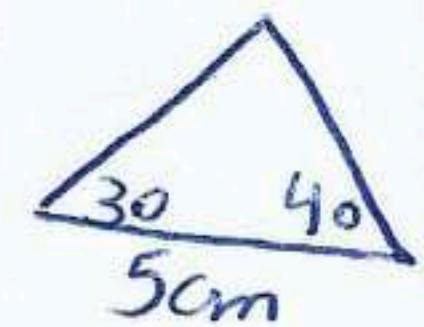
congruent



Congruent



Congruent



③ ④ mention two cases of congruent of two triangles

① two triangles are congruent if two sides and the included angle (S.A.S) of one triangle are congruent to the corresponding parts of the other triangle

② two triangles are congruent if each side of one triangle is congruent to the corresponding side of the other triangle (S.S.S)

b) In the opposite figure

prove that  $\triangle CBD \cong \triangle ABD$  and find  $m(\angle ABD)$

Solution

In  $\triangle CBD$  and  $\triangle ABD$

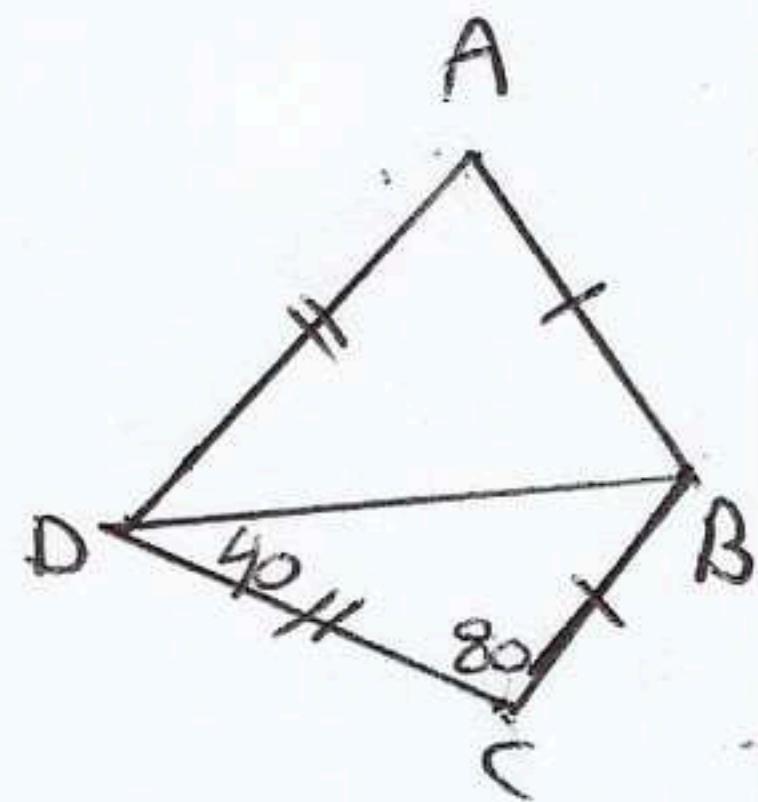
$$\begin{cases} AB = CB \\ \angle A = \angle C \end{cases}$$

$$\begin{cases} AD = CD \\ \angle A = \angle C \end{cases}$$

$DB$  is a common side (S)

Then  $\triangle CBD \cong \triangle ABD$

$$m(\angle ABD) = m(\angle CBD) = 180 - (40 + 80) = 60^\circ$$



④ ⑤ In the opposite figure find  $m(\angle C)$

is  $\overrightarrow{AB} \parallel \overrightarrow{CD}$

Solution

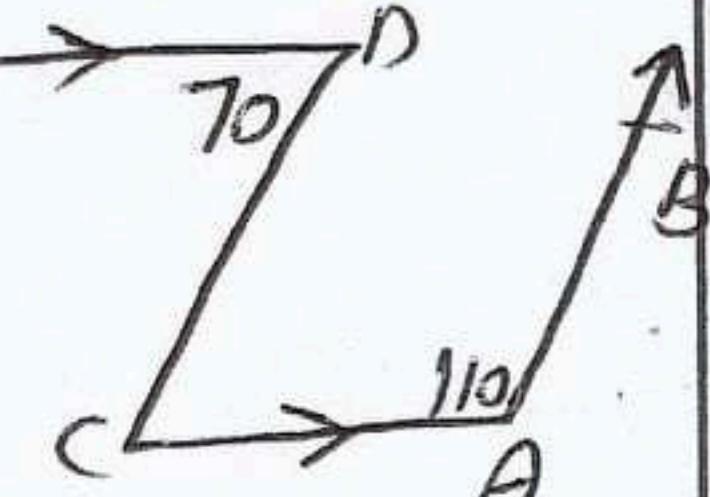
$\Rightarrow$  Since  $\overrightarrow{DE} \parallel \overrightarrow{CA}$  and  $\overrightarrow{DC}$  transversal to them

Then  $m(\angle C) = m(\angle D) = 70$  (Alternate angles) Eng / Jana Ahmed

$\Rightarrow$  Since  $m(\angle C) + m(\angle A) = 70 + 110 = 180$  two interior angles

Then  $\overrightarrow{AB} \parallel \overrightarrow{CD}$

16



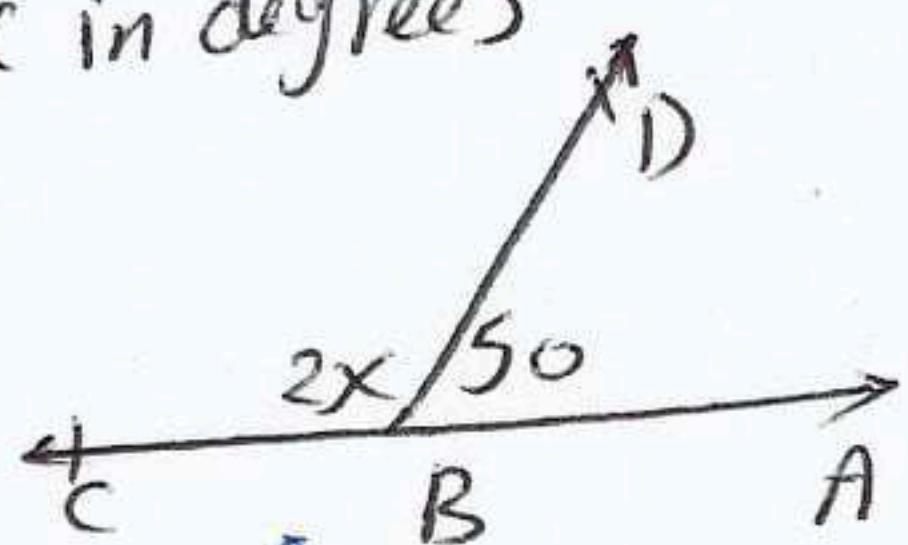
(b) using the geometric instruments draw  $\angle ABC$  where  $m(\angle B) = 80^\circ$  then draw  $\overrightarrow{BD}$  to bisect it (Don't remove the arcs try by yourself.

(5a) In the opposite figure find the value of  $x$  in degrees

Solution

$$m(\angle ABD) + m(\angle DBC) = 180^\circ$$

$$\text{Then } 2x = 180 - 50 = 130 \quad \text{then } x = \frac{130}{2} = 65^\circ$$



(b) In the opposite figure find  $m(\angle A)$  in degrees.

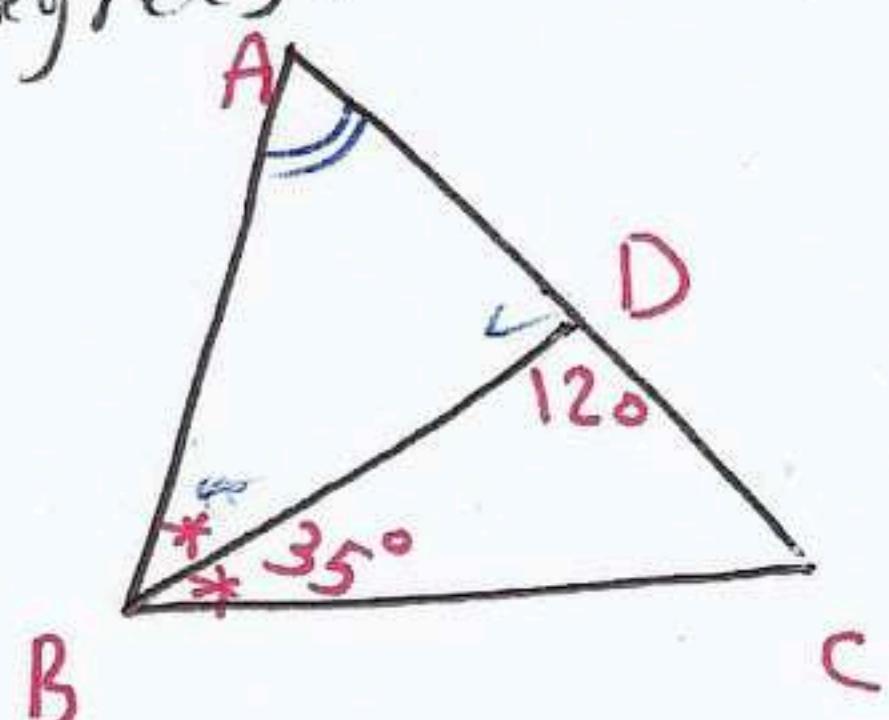
Solution

Since  $\overrightarrow{BD}$  bisects  $\angle ABC$

$$\text{Then } m(\angle ABD) = 35^\circ$$

$$m(\angle ADB) = 180^\circ - 120^\circ = 60^\circ$$

$$m(\angle A) = 180^\circ - (35^\circ + 60^\circ) = 85^\circ$$



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بال توفيق